

Craps Worksheet

Remember that according to the fundamental counting principle, there are 36 ways to roll two dice.

Examples:

Here are three worked out examples for you. They represent the cases of instant loss, instant win, and rolling a point and having to continue the game. Be sure to read carefully and see how the information fits into the table on the next page. Then use the table to find the probabilities of winning and of losing and the average length of a craps game.

First roll is a 3

- There are 2 ways to roll a sum of 3: 1-2 and 2-1. So the probability of getting a sum of 3 is $2/36$.
- If you roll a 3, you lose immediately, so the probability of winning is 0, the probability of losing is 1, and the probability of rolling again (for this game) is 0.
- Since the game is over with just that single roll, the total number of rolls is 1. The overall chance of winning is 0 and the chance of losing is 1.

First roll is a 7

- There are 6 ways to roll a sum of 7: 1-6, 2-5, 3-4, 4-3, 5-2, and 6-1. The probability of rolling a sum of 7 is therefore $6/36$.
- If you roll a 7, you win immediately, so the probability of winning is 1, the probability of losing is 0, and the probability of rolling again (for this game) is 0.
- Since the game is over with just that single roll, the total number of rolls was 1. The overall chance of winning is 1 and the chance of losing is 0.

First roll is a 4

- There are 3 ways to roll a sum of 4: 1-3, 2-2, and 3-1. Therefore, the probability of rolling a sum of 4 is $3/36$.
- A sum of 4 is a point. You continue to roll again until you either roll a 4 or a 7. If you roll a 4, you win and if you roll a 7, you lose. If you roll anything else, you roll again until you get the 4 or the 7. The probability of getting a 4 and winning is $3/36$ and the probability of getting a 7 and losing is $6/36$. Since the sum of the probabilities must be 1, the chance of rolling again is whatever is left out of the sum of 1. $1-3/36-6/36 = 27/36$.
- To find the average number of rolls before you roll a 4 (your point) or a 7, you can use an absorbing Markov chain. The transition matrix is shown below.

Transition Matrix when point is 4

	Win	Lose	Roll Again
Win	1	0	0
Lose	0	1	0
Roll Again	$3/36$	$6/36$	$27/36$

Since \mathbf{Q} is a 1×1 matrix, the \mathbf{I} (identity) matrix is really just the number one and the inverse of the matrix is just the reciprocal.

$$\mathbf{F} = (\mathbf{I} - \mathbf{Q})^{-1} = (1 - 27/36)^{-1} = (9/36)^{-1} = 36/9 = 4$$

The sum of the elements in each row of the \mathbf{F} matrix represents the total number of times you will spend in transient states (things other than your point or a 7) before entering an absorbing state (your point or a 7) and the game is over. Since there is only one element in the \mathbf{F} matrix, the sum is simply that number. Since the \mathbf{F} matrix is 4, which means it will take an average of 4 rolls before winning or losing the game. Since you had to roll once to get the four in the first place, the total number of rolls would be 5.

- To find the probability of winning or losing, find the \mathbf{FR} matrix.

$$\mathbf{FR} = (36/9) \cdot \begin{matrix} & \begin{matrix} win & lose \end{matrix} \\ \begin{matrix} win & lose \end{matrix} & \begin{bmatrix} 3/36 & 6/36 \end{bmatrix} \end{matrix} = \begin{bmatrix} 1/3 & 2/3 \end{bmatrix}$$

The elements in the \mathbf{FR} matrix represent the long term probability of going from a transient state to an absorbing state. Since the first column is winning and the second column is losing, the probabilities of winning and losing when your point is a 4 are $1/3$ and $2/3$.

Table of Probabilities and Number of Rolls

Sum	1 st roll prob.	Any other* roll			Overall results		
		Win	Lose	Again	Rolls	Win	Lose
2							
3	2/36	0	1	0	1	0	1
4	3/36	3/36	6/36	27/36	5	1/3	2/3
5							
6							
7	6/36	1	0	0	1	1	0
8							
9							
10							
11							
12							

* the table says "other" roll, but for the 2, 3, 7, 11, and 12, there is only one roll.

Overall Results

- The probabilities of winning and losing can be found by using the general multiplication principle. Multiply the probability of getting a particular sum on the first roll by the probability of winning (or losing) the game if that sum occurs and then add them together to get the overall probability.
- The average length of a game is the expected value of the total number of rolls. Multiply the number of rolls for each sum by the probability of getting that sum and add them together.

Probability of Winning	Probability of Losing	Average length of a game of craps