Mathematical Notation

Math 160 - Finite Mathematics

Name : ________________________

Use Word or WordPerfect to recreate the following documents. Each article is worth 10 points and can be printed and given to the instructor or emailed to the instructor at james@richland.edu. If you use Microsoft Works to create the documents, then you must print it out and give it to the instructor as he can't open those files.

Type your name at the top of each document.

Include the title as part of what you type. The lines around the title aren't that important, but if you will type ----- at the beginning of a line and hit enter, both Word and WordPerfect will draw a line across the page for you.

For expressions or equations, you should use the equation editor in Word or WordPerfect. The documents were created using a 14 pt Times New Roman font with standard 1" margins. The equations were created using 16 pt font, but feel free to use a smaller font.

For individual symbols (μ, σ, etc), you can insert symbols. In Word, use "Insert / Symbol" and choose the Symbol font. For WordPerfect, use Ctrl-W and choose the Greek set.

There are instructions on how to use the equation editor in a separate document. Be sure to read through the help it provides. There are some examples at the end that walk students through the more difficult problems. You will want to read the handout on using the equation editor if you have not used this software before.

These notations are due at the beginning of class on the day of the exam for that chapter. That is, the chapter 3 notation is due on the day of the chapter 3 test. Late work will be accepted but will lose 10% per class period.

Chapter 3 - Finance

Simple Interest

$$I = PRT$$

I = Interest
P = Principal
R = Rate
T = Time

Compound Interest

$$A = P\left(1+i\right)^{n}$$

A = Amount
P = Principal
i = Periodic rate
n = Number of periods

Future Value Annuities

$$FV = PMT\left(\frac{\left(1+i\right)^{n}-1}{i}\right)$$

FV = Future value
PMT = Payment
i = Periodic rate
n = Number of periods

Present Value Annuities

$$PMT = PV\left(\frac{i}{1-\left(1+i\right)^{-n}}\right)$$

PV = Present value
PMT = Payment
i = Periodic rate
n = Number of periods

Chapter 4 - Systems of Equations, Matrices

Matrix Addition

$$\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix} + \begin{bmatrix} 5 & -2 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 7 & -1 \\ 4 & 5 \end{bmatrix}$$

Determinant of a matrix

$$\begin{vmatrix} 2 & 1 & 3 \\ -1 & 2 & 4 \\ 5 & 1 & -6 \end{vmatrix} = -51$$

Augmented matrix in reduced row-echelon form

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & -3 \end{array}\right]$$

Solving a system of linear equations using matrix inverses

$$\mathbf{AX} = \mathbf{B} \Rightarrow \mathbf{X} = \mathbf{A}^{-1}\mathbf{B}$$

Leontief Input-Output Model

$$\mathbf{X} = \left(\mathbf{I} - \mathbf{M}\right)^{-1} \mathbf{D}$$

X = Output matrix
M = Technology Matrix
D = Demand Matrix

Chapter 5 - Linear Programming

Fundamental Theorem of Linear Programming
If there is a solution to a linear programming problem, then it will occur at one or more corner points of the feasible region or on the boundary between two corner points.

Hint, place following two systems into a matrix without brackets. Define the matrix row spacing to be 100% and the matrix column spacing to be 50%. Don't type this hint.

System of linear inequalities

$$\begin{array}{rcrcr} 3x & + & 4y & \le & 36 \\ 3x & + & 2y & \le & 30 \\ & & x & \ge & 0 \\ & & y & \ge & 0 \end{array}$$

Initial system for a standard maximization problem

$$\begin{array}{rcrcrcccccr} x_1 & + & x_2 & + & x_3 & + & s_1 & & & & = 100 \\ 40x_1 & + & 20x_2 & + & 30x_3 & & & + s_2 & & & = 3200 \\ x_1 & + & 2x_2 & + & x_3 & & & & + s_3 & & = 160 \\ -100x_1 & - & 300x_2 & - & 200x_3 & & & & & + P & = 0 \end{array}$$

Be sure to reset the matrix row spacing to 150% and the matrix column spacing to 100%.

Initial tableau for a standard maximization problem

$$\left[\begin{array}{rrrrr|r} 1 & 2 & 1 & 0 & 0 & 32 \\ 3 & 4 & 0 & 1 & 0 & 84 \\ \hline -50 & -80 & 0 & 0 & 1 & 0 \end{array}\right]$$

Chapter 6 - Sets and Counting

If **A** = { 1, 2, 4, 6 } and **B** = { 2, 3, 5 }, then ...

... the union of the sets is $\mathbf{A} \bigcup \mathbf{B} = \{1, 2, 3, 4, 5, 6\}$

... the intersection of the sets is $\mathbf{A} \bigcap \mathbf{B} = \{2\}$

Fundamental Counting Principle

The total number of ways that two events can happen is found by multiplying together the number of ways that each event can happen.

Permutations

A permutation is an arrangement of objects without repetition but with regard to order.

$${}_nP_r = \frac{n!}{(n-r)!}$$

Combinations

A combination is an arrangement of objects without repetition and without regard to order.

$${}_nC_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}$$

Chapter 7 - Probability

Probability formulas

Addition Rule

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Multiplication Rule

$$P(A \cap B) = P(A)P(B \mid A)$$

Conditional Probability

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

Complement of an Event

$$P(E') = 1 - P(E)$$

Expected value

$$E(x) = \sum_{k=1}^{n} x_k p(x_k)$$

Decision Theory

Expected value (Bayesian) criterion. Find the expected value under each action and choose the action with the largest expected value.

Maximax criterion. Find the maximum payoff under each action and then choose the action with the largest best case scenario.

Maximin criterion. Find the minimum payoff under each action and then choose the action with the largest worst case scenario.

Minimax criterion. Find the opportunistic loss for each state of nature. Then find the maximum opportunistic loss for each action and choose the action with the smallest maximum loss.

Chapter 8 - Statistics

Statistics Formulas

$$\overline{x} = \frac{1}{n}\sum_{k=1}^{n} x_k$$

$$s^2 = \frac{\sum_{k=1}^{n}\left(x_k - \overline{x}\right)^2}{n-1}$$

Binomial Probabilities

A binomial experiment is a fixed number of independent trials each having exactly two possible outcomes.

$$P\left(x\right) = \binom{n}{x} p^x q^{n-x}; \quad p + q = 1$$

n = total number of trials
p = probability of success on a single trial
q = probability of failure on a single trial
x = number of successes out of n trials.

Z-scores

A standardized score is found by taking the value, subtracting the mean, and dividing by the standard deviation.

$$z = \frac{x - \mu}{\sigma}$$

Chapter 9 - Game Theory

Assume that the game matrix is $\mathbf{M} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

The solution to a two player, zero-sum, non-strictly determined game is

$$\mathbf{P}^* = \begin{bmatrix} \dfrac{d-c}{D} & \dfrac{a-b}{D} \end{bmatrix}$$

$$\mathbf{Q}^* = \begin{bmatrix} \dfrac{d-b}{D} \\ \dfrac{a-c}{D} \end{bmatrix}$$

where $D = (a+d)-(b+c)$

Linear Programming Problem

Assume that **M** has all positive entries.

The optimal row player solution is found by solving this linear programming problem

Minimize: $z = \dfrac{1}{v} = x_1 + x_2$ where $x_1 = \dfrac{p_1^*}{v}$ and $x_2 = \dfrac{p_2^*}{v}$

Subject to:
$$\begin{array}{ccccc} ax_1 & + & cx_2 & \geq & 1 \\ bx_1 & + & dx_2 & \geq & 1 \\ x_1 & , & x_2 & \geq & 0 \end{array}$$

The optimal column player solution is found by solving this linear programming problem.

Maximize: $z = \dfrac{1}{v} = y_1 + y_2$ where $y_1 = \dfrac{q_1^*}{v}$ and $y_2 = \dfrac{q_2^*}{v}$

Subject to:
$$\begin{array}{ccccc} ay_1 & + & by_2 & \leq & 1 \\ cy_1 & + & dy_2 & \leq & 1 \\ y_1 & , & y_2 & \geq & 0 \end{array}$$

Chapter 10 - Markov Chains

S is a state matrix, **P** is the transition matrix

Regular Markov chains

State Matrices

$$\mathbf{S}_1 = \mathbf{S}_0\mathbf{P}$$

$$\mathbf{S}_2 = \mathbf{S}_1\mathbf{P} = \mathbf{S}_0\mathbf{P}^2$$

$$\mathbf{S}_3 = \mathbf{S}_2\mathbf{P} = \mathbf{S}_0\mathbf{P}^3$$

Steady State Matrix

$$\mathbf{S} = \mathbf{SP}$$

If the Markov chain is regular, then there is a limiting matrix $\overline{\mathbf{P}} = \mathbf{P}^\infty$ where each row is the Steady State matrix.

Absorbing Markov Chains

$$\mathbf{P} = \left[\begin{array}{c|c} \mathbf{I} & \mathbf{0} \\ \hline \mathbf{R} & \mathbf{Q} \end{array}\right]$$

Fundamental Matrix F

The element in row R, column C of the fundamental matrix represents the expected number of times you will spend in state transient C of the system if you start in transient state R before entering an absorbing state.

$$\mathbf{F} = \left(\mathbf{I} - \mathbf{Q}\right)^{-1}$$

Limiting Matrix

$$\overline{\mathbf{P}} = \mathbf{P}^\infty = \left[\begin{array}{c|c} \mathbf{I} & \mathbf{0} \\ \hline \mathbf{FR} & \mathbf{0} \end{array}\right]$$

The element in row R, column C of the matrix $\mathbf{FR}$ represents the long term probability of ending up in absorbing state C if you started in transient state R.