Finding the domain of the composition of a function with another one.

Suppose \( f \) and \( g \) are functions as follows:

\[
\begin{array}{c|cccc}
 x & -1 & 0 & 2 & 3 & 5 \\
 \hline
 f(x) & 0 & -1 & 5 & 2 & -2 \\
 g(x) & 1 & 3 & 6 & -1 & 2 \\
\end{array}
\]

Note that the domain of these 2 functions is the same: \( \{-1, 0, 2, 3, 5\} \).

We would like to find the domains of their compositions.

1. \( f \circ g = f[g(x)] \)
   a. \( f \circ g(-1) = f[g(-1)] = f(1) \), but there is no output for it. Therefore \( f \circ g(-1) \) is undefined.
   b. \( f \circ g(0) = f[g(0)] = f(3) = 2 \)
   c. \( f \circ g(2) = f[g(2)] = f(6) \text{ undefined} \)
   d. \( f \circ g(3) = f[g(3)] = f(-1) = 0 \)
   e. \( f \circ g(5) = f[g(5)] = f(2) = 5 \)

Now we can find the domain: \( D_{f \circ g} : \{0, 3, 5\} \).

2. \( g \circ f = g[f(x)] \)
   a. \( g \circ f(-1) = g[f(-1)] = g(0) = 3 \)
   b. \( g \circ f(0) = g[f(0)] = g(-1) = 1 \)
   c. \( g \circ f(2) = g[f(2)] = g(5) = 2 \)
   d. \( g \circ f(3) = g[f(3)] = g(2) = 6 \)
   e. \( g \circ f(5) = g[f(5)] = g(-2) \text{ undefined} \)

\( D_{g \circ f} : \{-1, 0, 2, 3\} \).