Euler (1707-1783) came up with the following identity which is probably the most important formula in mathematics. Calculus students can prove this identity with a little effort.

\[ e^{i\theta} = \cos \theta + i \sin \theta, \quad \text{where} \quad i = \sqrt{-1} \]

From this identity we can generate other formulas like:

\[ \cos \theta = \frac{e^{i\theta} + e^{-i\theta}}{2} \quad \text{and} \quad \sin \theta = \frac{e^{i\theta} - e^{-i\theta}}{2i} \]

Calculus students should quickly notice the similarity of the form of these two functions with the hyperbolic functions \( \cosh \theta \) and \( \sinh \theta \).

In the Euler’s formula if we replace \( \theta = \pm \pi \) and \( \pm \frac{\pi}{2} \) we will see interesting numbers like:

\[ e^{\pm \pi i} = \cos (\pm \pi) + i \sin (\pm \pi) = -1 \] (since \( \cos \theta \) is an even function)

\[ e^{\pm \frac{\pi}{2} i} = \cos \left( \pm \frac{\pi}{2} \right) + i \sin \left( \pm \frac{\pi}{2} \right) = \pm i \] (since \( \sin \theta \) is an odd function).

Another interesting number related to \( i \) is: \( i^i = e^{-\frac{\pi}{2}} \). This equality, which is very unique, relates an imaginary number to power of itself to the most important irrational number to power of a popular irrational number.

Students in calculus, with a lot of manipulations, should be able to show the way to get to this equation.

Even though the next equation has no direct relation with our imaginary number, I would like you to see \( \pi \) in terms of many 2s.

\[ \frac{2}{\pi} = \frac{\sqrt{2}}{2}, \frac{\sqrt{2 + \sqrt{2}}}{2}, \frac{\sqrt{2 + \sqrt{2 + \sqrt{2}}}}{2}, \ldots \]

Students in calculus (with a little help) should be able to derive this identity.

Now, you should be able to connect the right side of the last identity with the \( i \), and create a few interesting numbers of your own.

Good luck.