Mathematical Notation

Math 121 - Calculus & Analytic Geometry I

Name : ____________________________

Use Word or WordPerfect to recreate the following documents. Each article is worth 10 points and can be printed and given to the instructor or emailed to the instructor at james@richland.edu. If you use Microsoft Works to create the documents, then you must print it out and give it to the instructor as he can't open those files.

Type your name at the top of each document.

Do not create the watermark $f\left(x\right)=\int_{-\infty}^{x}\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}\left(\frac{t-\mu}{\sigma}\right)^2}dt$ on your document.

This is in there so you don't just photocopy the document and give it back to me.

For expressions or equations, you should use the equation editor in Word or WordPerfect. The documents were created using a 14 pt Times New Roman font with standard 1" margins.

For individual symbols (μ, σ, etc), you can insert symbols. In Word, use "Insert / Symbol" and choose the Symbol font. For WordPerfect, use Ctrl-W and choose the Greek set.

The due date for each of these documents is the day after the exam for that chapter. While the material is not due until after the exam, it is recommended that you create it ahead of time because the material will help you review for the exam.

Chapter 1 - Trigonometry Review

Degrees	$0°$	$30°$	$45°$	$60°$	$90°$
Radians	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$\sin\theta$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
$\tan\theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	undef

$$\sin^2 x + \cos^2 x = 1 \quad 1 + \tan^2 x = \sec^2 x \quad \cot^2 x + 1 = \csc^2 x$$

$$\sin\left(\frac{\pi}{2} - x\right) = \cos x \quad \cos\left(\frac{\pi}{2} - x\right) = \sin x \quad \tan\left(\frac{\pi}{2} - x\right) = \cot x$$

$$\cos(a \pm b) = \cos a \cos b \mp \sin a \sin b$$

$$\sin(a \pm b) = \sin a \cos b \pm \cos a \sin b$$

$$\tan(a \pm b) = \frac{\tan a \pm \tan b}{1 \mp \tan a \tan b}$$

$$\cos 2x = \cos^2 x - \sin^2 x \quad \sin 2x = 2 \sin x \cos x \quad \tan 2x = \frac{2 \tan x}{1 - \tan^2 x}$$

$$\cos^2 x = \frac{1 + \cos 2x}{2} \quad \sin^2 x = \frac{1 - \cos 2x}{2}$$

Chapter 2 - Limits

When finding a finite limit, simply substitute the value into the expression unless it causes problems.

The two sided limit $\lim\limits_{x\to a} f\left(x\right)$ exists if and only if both one sided limits $\lim\limits_{x\to a^+} f\left(x\right)$ and $\lim\limits_{x\to a^-} f\left(x\right)$ exist and are equal to each other.

If a rational function has a limit of the form 0/0, then there is a common factor in both the numerator and the denominator. Factor both, reduce, and then evaluate the limit.

When finding infinite limits of polynomial and rational functions, only the leading term needs to be considered. This is only true for limits as $x \to +\infty$ or $x \to -\infty$. That is ...

$$\lim_{x\to\infty}\left(a_n x^n + a_{n-1}x^{n-1} + \cdots + a_0\right) = \lim_{x\to\infty}\left(a_n x^n\right)$$

$$\lim_{x\to\infty}\left(\frac{a_n x^n + a_{n-1}x^{n-1} + \cdots + a_0}{b_m x^m + b_{m-1}x^{m-1} + \cdots + b_0}\right) = \lim_{x\to\infty}\left(\frac{a_n x^n}{b_m x^m}\right)$$

$\lim\limits_{x\to a} f\left(x\right) = L$ if $\forall \varepsilon > 0,\ \exists \delta > 0 \ni \left|f\left(x\right) - L\right| < \varepsilon$ whenever $0 < \left|x - a\right| < \delta$.

$$\lim_{x\to 0}\frac{\sin x}{x} = 1 \quad \lim_{x\to 0}\frac{1-\cos x}{x} = 0 \quad \lim_{x\to 0}\frac{\tan x}{x} = 1$$

A function *f* is continuous at $x = a$ if 1) $f\left(a\right)$ is defined, 2) $\lim\limits_{x\to a} f\left(x\right)$ exists, and 3) $\lim\limits_{x\to a} f\left(x\right) = f\left(a\right)$.

If *f* is continuous on [a,b] and *k* is between $f\left(a\right)$ and $f\left(b\right)$, then there exists at least one $x \in \left[a,b\right]$ such that $f\left(x\right) = k$.

Chapter 3 - Derivatives

Notation

$$\frac{d}{dx}\left[f(x)\right] = f'(x) = D_x\left[f(x)\right] = \frac{dy}{dx} = y'$$

$$\frac{d^2}{dx^2}\left[f(x)\right] = f''(x) = D_{xx}\left[f(x)\right] = \frac{d^2y}{dx^2} = y''$$

Definition

$$f'(x) = \frac{d}{dx}\left[f(x)\right] = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

Power Rule

$$\frac{d}{dx}\left[x^n\right] = n\cdot x^{n-1}$$

Product Rule

$$\left[f\cdot g\right]' = f\cdot g' + f'\cdot g$$

Quotient Rule

$$\left(\frac{f}{g}\right)' = \frac{g\cdot f' - f\cdot g'}{g^2}$$

Chain Rule

$$\left[f\left(g(x)\right)\right]' = f'\left(g(x)\right)\cdot g'(x)$$

$$\frac{dy}{dx} = \frac{dy}{du}\cdot\frac{du}{dx}$$

Trigonometric Functions

$$\frac{d}{dx}[\sin x] = \cos x \qquad \frac{d}{dx}[\tan x] = \sec^2 x \qquad \frac{d}{dx}[\sec x] = \sec x\tan x$$

$$\frac{d}{dx}[\cos x] = -\sin x \qquad \frac{d}{dx}[\cot x] = -\csc^2 x \qquad \frac{d}{dx}[\csc x] = -\csc x\cot x$$

Local Linear Approximation

$$f(x) \approx f(x_0) + f'(x_0)\cdot\Delta x$$

Chapter 4 - Applications of the Derivative

If *f* is differentiable, then *f* is increasing when $f'(x) > 0$, decreasing when $f'(x) < 0$, and constant when $f'(x) = 0$.

Critical points occur where $f'(x) = 0$ or $f'(x)$ is undefined. Stationary points are the critical points where $f'(x) = 0$.

If *f* is twice differentiable, then *f* is concave up when $f''(x) > 0$ and concave down when $f''(x) < 0$.

Inflection points occur when concavity changes. This can occur when $f''(x) = 0$ or $f''(x)$ is undefined.

Relative extrema can only occur at critical points.

If *f* is twice differentiable at $x = a$ and $f'(a) = 0$, then there will be a relative minimum at $x = a$ if $f''(a) > 0$ and a relative maximum at $x = a$ if $f''(a) < 0$. If $f''(a) = 0$, the second derivative test is inconclusive.

Rectilinear Motion

Position	$s(t)$
Velocity	$v(t) = s'(t) = \dfrac{ds}{dt}$
Speed	$speed = \lvert v(t) \rvert = \left\lvert \dfrac{ds}{dt} \right\rvert$
Acceleration	$a(t) = v'(t) = \dfrac{dv}{dt} = s''(t) = \dfrac{d^2s}{dt^2}$

Chapter 5 - Integration

$$\frac{d}{dx}\left[\int f(x)\,dx\right] = f(x)$$

$$\int x^n\,dx = \tfrac{1}{n+1}x^{n+1} + C$$

$$\sum_{k=1}^{n}(a_k \pm b_k) = \sum_{k=1}^{n}a_k \pm \sum_{k=1}^{n}b_k$$

$$\int_a^b f(x)\,dx = \lim_{\max \Delta x_k \to 0}\sum_{k=1}^{n} f(x_k^*)\Delta x_k$$

$$\int_a^a f(x)\,dx = 0$$

$$\int_b^a f(x)\,dx = -\int_a^b f(x)\,dx$$

$$\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$$

If *f* is continuous on [a,b] and *F* is any antiderivative of *f* on [a,b], then

$$\int_a^b f(x)\,dx = F(x)\Big]_a^b = F(b) - F(a)$$

If *f* is continuous and $F(x) = \int_a^x f(t)\,dt$ is an antiderivative of *f*, then

$$\frac{d}{dx}\left[\int_a^x f(t)\,dt\right] = f(x)$$

$$f_{ave} = \frac{1}{b-a}\int_a^b f(x)\,dx$$

Chapter 6 - Applications of Integration

Area between two curves

$$A = \int_a^b \left[f(x) - g(x) \right] dx$$

Volume of solid of revolution by disk method (x-axis)

$$V = \int_a^b \pi \left[f(x) \right]^2 dx$$

Volume of solid of revolution by washer method (x-axis)

$$V = \int_a^b \pi \left(\left[f(x) \right]^2 - \left[g(x) \right]^2 \right) dx$$

Volume of solid of revolution by cylindrical shell method (y-axis)

$$V = \int_a^b 2\pi \cdot x \cdot f(x) dx$$

Length of plane curve

$$L = \int_a^b \sqrt{\left(\frac{dx}{dt} \right)^2 + \left(\frac{dy}{dt} \right)^2} dt$$

Area of a surface of revolution (x-axis)

$$S = \int_a^b 2\pi \cdot f(x) \sqrt{1 + \left[f'(x) \right]^2} dx$$

Work

$$W = \int_a^b F(x) dx$$

Fluid Force

$$F = \int_a^b \rho \cdot h(x) \cdot w(x) dx$$