Exam 2 - Chapter 8 Math 122 Name : _____
70 Points - Take Home Calculus & Analytic Geometry II

1. Find the average water level on the Sangamon River at Illinois Rt 48 in Decatur, IL, for Sept 2002. (10 points).

Collect the water level data from the US Geological Society website (URL given below) and record it on the attached sheet. (5 points)

From the web page, request the "stage" data for 30 days in a tab-delimited text data file. Although the output includes data at 15 minute intervals, only record the midnight [24:00:00] and noon [12:00:00] readings.

Sample Output

Date and Time (CST)	Stage
2002.08.31 24:00:00	3.52
2002.09.01 12:00:00	3.57
2002.09.01 24:00:00	3.38
2002.09.02 12:00:00	3.39
2002.09.02 24:00:00	2.98
2002.09.03 12:00:00	2.60
2002.09.03 24:00:00	2.38
2002.09.04 12:00:00	2.63
2002.09.04 24:00:00	2.32
2002.09.05 12:00:00	2.30
2002.09.05 24:00:00	2.46
2002.09.06 12:00:00	2.43
2002.09.06 24:00:00	2.40
2002.09.07 12:00:00	2.39
2002.09.07 24:00:00	2.38

Record the midnight readings from 9/1/2002 [2002.08.31 24:00:00] through 10/1/2002 [2002.09.30 24:00:00] so that you have a full 30 days of information (but 31 endpoints). Also record the readings at noon from 9/1/2002 [2002.09.01 12:00:00] to 9/30/2002 [2002.09.30 12:00:00].

The first week's worth of data is shown in the table above, but you will need to gather the rest of the information from the Internet. The site only displays information for the last 30 days, but with the data above, you should be able to get it all before the test is due.

http://www-il.usgs.gov/nwis-w/IL/data.components/rt.cgi?statnum=05573540

a. Using the 31 values recorded at midnight, approximate the average water stage using (4 points)

   i. the left-hand endpoints

   ii. the right-hand endpoints

   iii. the trapezoidal method

   iv. Simpson's method

b. Using the 30 values recorded at noon, approximate the average water stage using the midpoints. (1 point)

2. The Laplace transform, $\mathcal{L}\{f(x)\}$, is useful in differential equations. Find the following Laplace transforms. Note that the Laplace Transform is a function of *s* only, there is no *x* in the transform. Look at problems 8.8.56-57 in the book for additional information. The restrictions on *s* that are given in the book are necessary so that the improper integral converges, be sure to state proper restrictions. Most of the integrals will require integration by parts if done by hand, you may use a table of integrals. An example problem has been worked for you.

$$\mathcal{L}\{f(x)\} = \int_0^{+\infty} e^{-sx} f(x)dx$$

Work any five (5) of the following transforms. (10 points)

a. $\mathcal{L}\{a\}$

b. $\mathcal{L}\{ax\}$

c. $\mathcal{L}\{\sin ax\}$

d. $\mathcal{L}\{\cos ax\}$

e. $\mathcal{L}\{\sinh ax\}$

f. $\mathcal{L}\{\cosh ax\}$

g. $\mathcal{L}\{xe^{-ax}\}$

Example: Find $\mathcal{L}\{e^{ax}\}$

$$= \int_0^{+\infty} e^{-sx}e^{ax}dx = \int_0^{+\infty} e^{-(s-a)x}dx$$

let $u = -(s-a)x$, $du = -(s-a)dx$

$u\big|_{x\to+\infty} = -\infty$ *if* $s > a$, $u\big|_{x=0} = 0$

$$= \frac{-1}{s-a}\int_0^{-\infty} e^u du = \frac{1}{s-a}\int_{-\infty}^{0} e^u du$$

$$= \frac{e^u}{s-a}\Big|_{-\infty}^{0} = \frac{1}{s-a}\left(e^0 - \lim_{b\to-\infty} e^b\right)$$

$$= \frac{1-0}{s-a} = \frac{1}{s-a}, \; s > a$$

3. Use a table of integrals to find the integral. In each case, copy the number of the formula and the integration formula itself. Then give the values of any variables (ex: *a* or *u*). Finally, find the integral and simplify. (10 points)

a. Problem 8.6.10

b. Problem 8.6.20

c. Problem 8.6.24

d. Problem 8.6.30

e. Problem 8.6.38

4. The Gamma function, $\Gamma(x)$, is defined as shown. This integral converges if and only if $x > 0$.

$$\Gamma(x) = \int_0^{+\infty} t^{x-1} e^{-t}\, dt$$

a. Find $\Gamma(1)$. (2 points)

b. Use integration by parts to prove that $\Gamma(x+1) = x\Gamma(x)$ for all $x > 0$. (3 points)

c. Use the results of part a. and b. to find the following. (4 points)

i. $\Gamma(2)$

ii. $\Gamma(3)$

iii. $\Gamma(4)$

iv. $\Gamma(5)$

d. Make a conjecture about $\Gamma(n)$ for positive values of n. (1 point)

5. Use a computer algebra system (Derive, Maple, Mathematica, TI-89/92) to find the following integrals. Use the equation editor to answer this question. Type the original problem and the solution in the form "problem = solution". (10 points)

a. Problem 8.6.42

b. Problem 8.6.50

c. Problem 8.6.56

d. Problem 8.6.62

e. Problem 8.6.70

6. Work ten (10) of the following problems by hand. There must be exactly two problems from each section (8.2, 8.3, 8.4, 8.5, and 8.8). Show all work. Clearly identify the section number and problem. Attach the problems in section order. You may use the reduction formulas where necessary, but otherwise do not use the table of integrals. You may use a CAS to check your answer, but show work. (20 points)

a. Problem 8.2.8

b. Problem 8.2.20

c. Problem 8.2.38

d. Problem 8.3.8

e. Problem 8.3.30

f. Problem 8.3.44

g. Problem 8.4.12

h. Problem 8.4.18

i. Problem 8.4.40

j. Problem 8.5.14

k. Problem 8.5.20

l. Problem 8.5.26

m. Problem 8.8.8

n. Problem 8.8.22

o. Problem 8.8.42

Water Level on Sangamon River at Decatur, IL, for September, 2002.

#	Date	Midnight Reading	Noon Reading	#	Date	Midnight Reading	Noon Reading
1	01-Sep	3.52	3.57	17	17-Sep		
2	02-Sep	3.38	3.39	18	18-Sep		
3	03-Sep	2.98	2.60	19	19-Sep		
4	04-Sep	2.38	2.63	20	20-Sep		
5	05-Sep	2.32	2.30	21	21-Sep		
6	06-Sep	2.46	2.43	22	22-Sep		
7	07-Sep	2.40	2.39	23	23-Sep		
8	08-Sep	2.38		24	24-Sep		
9	09-Sep			25	25-Sep		
10	10-Sep			26	26-Sep		
11	11-Sep			27	27-Sep		
12	12-Sep			28	28-Sep		
13	13-Sep			29	29-Sep		
14	14-Sep			30	30-Sep		
15	15-Sep			31	01-Oct		
16	16-Sep						