

Mathematical Notation

Math 116 - College Algebra

Name : _____

Use Word or WordPerfect to recreate the following documents. Each article is worth 10 points and can be printed and given to the instructor or emailed to the instructor at james@richland.edu. If you use Microsoft Works to create the documents, then you must print it out and give it to the instructor as he can't open those files.

Type your name at the top of each document.

For expressions or equations, you should use the equation editor in Word or WordPerfect. The documents were created using a 14 pt Times New Roman font with standard 1" margins.

For individual symbols (μ , σ , etc), you can insert symbols. In Word, use "Insert / Symbol" and choose the Symbol font. For WordPerfect, use Ctrl-W and choose the Greek set.

The due date for each of these documents is the day of the exam for that chapter. Late work will be accepted but will lose 10% per class period.

There are periodic notes about how the expression was created. You do not need to put those notes on your paper.

Chapter 1 - Shifting, Reflecting, and Stretching Graphs

Consider the function defined by $g(x) = a \cdot f\left(\frac{x-c}{b}\right) + d$.

The shifting transformation is a translation where the graph retains its size and shape but its position is changed. Shifts occur because of addition or subtraction, so the two variables in the above function that would cause shifts are c and d .

A scaling transformation is one where the size of the graph is changed. Because the size is changed, the position is often changed with it. Scaling occurs because of multiplication or division, so the two variables in the above function that would cause scaling are a and b .

A reflection is a transformation is a special case of scaling where the constant is -1 . The graph retains its size and shape but is reflected about an axis.

The constants grouped with the x (b and c) are horizontal transformations, that is, they affect the x but not the y . Also notice that they act backwards from what you might think (hence the subtraction and division). The constants that are not grouped with the x (a and d) affect the y but not the x . They are vertical transformations and act the way you would expect.

The following chart illustrates some examples.

Graph	Translation	Domain	Range
$y = f(x)$	None	$[3, 6)$	$(1, 5]$
$y = 3f(x - 2)$	Multiply y's by 3, Right 2	$[5, 8)$	$(3, 15]$
$y = 2 - f(3x + 6)$	Reflect about the x-axis, up 2, divide x's by 3, left 2	$[-1, 0)$	$[-3, 1)$
$y = f^{-1}(x) + 3$	Switch the x's and y's, up 3	$(1, 5]$	$[6, 9)$

Chapter 2 - Solving Equations Algebraically

Complete the square to solve $x^2 - 4x + 7 = 0$.

$$\begin{aligned}x^2 - 4x + 7 &= 0 \\x^2 - 4x &= -7 \\x^2 - 4x + 4 &= -7 + 4 \\(x - 2)^2 &= -3 \\x - 2 &= \pm\sqrt{-3} \\x &= 2 \pm i\sqrt{3}\end{aligned}$$

Note: the line spacing was changed from 150% to 100% for this chapter and the equations are aligned at the = sign. Don't type this note on your paper.

When solving an equation involving radicals, isolate the radical and then square both sides. Be sure to check for extraneous solutions.

$$\begin{aligned}\sqrt{x+1} - 3x &= 1 \\\sqrt{x+1} &= 3x + 1 \\(\sqrt{x+1})^2 &= (3x + 1)^2 \\x + 1 &= 9x^2 + 6x + 1 \\9x^2 + 5x &= 0 \\x(9x + 5) &= 0\end{aligned}$$

There are two answers, $x = 0$ and $x = -5/9$, but only $x = 0$ checks. The answer is $x = 0$.

When factoring, always factor out the greatest common factor first. The greatest common factor is the factor with the smallest exponents.

$$\begin{aligned}3x(x-1)^{1/2} + 2(x-1)^{3/2} &= 0 \\(x-1)^{1/2}[3x + 2(x-1)] &= 0 \\(x-1)^{1/2}(5x-2) &= 0\end{aligned}$$

There are two answers, $x = 1$ and $x = 2/5$, but only $x = 1$ is in the domain. The one-half power means the square root, so there is really a $\sqrt{x-1}$ in the problem and the implied domain is $x \geq 1$. Since $x = 2/5$ doesn't fall in that domain, it can't be used as an answer. The answer is $x = 1$.

Chapter 3 - Polynomials

Consider the polynomial $f(x) = -2(x + 3)^2(x - 5)^3(x + 4)$.

If it were multiplied out, the leading term would be $-2 \cdot x^2 \cdot x^3 \cdot x = -2x^6$ and the constant would be $f(0) = -2(3)^2(-5)^3(4) = 9000$.

Because the degree of the polynomial is 6, there will be exactly 6 real or complex roots and a maximum of 5 turns.

On the far right side of the graph, as $x \rightarrow +\infty$, $y \rightarrow -\infty$ because the leading coefficient is negative. On the far left side of the graph, as $x \rightarrow -\infty$, $y \rightarrow -\infty$ because the degree is even so the left side will do the same as the right side.

The rational root theorem says that any rational roots will be of the form of a factor of 9000 over a factor of 2.

The graph of the function will cross the x -axis as $x = 5$ and $x = -4$ because the corresponding factors have odd exponents. The graph will touch, but not cross, the x -axis at $x = -3$ because the corresponding factor has an even exponent.

When the coefficients are real, then complex solutions come in pairs. If $a + bi$ is a root, then so is $a - bi$.

When the coefficients are rational, then irrational solutions involving square roots come in pairs. If $a + \sqrt{b}$ is a root, then so is $a - \sqrt{b}$.

Chapter 4 - Exponential and Logarithmic Functions

A logarithm is an exponent.

Conversion between exponential form and logarithmic form

$$x = a^y \Leftrightarrow y = \log_a x$$

Properties of Logarithms

$$\log_a 1 = 0$$

$$\log_a a = 1$$

$$\log_a a^x = x$$

$$a^{\log_a x} = x, \quad x > 0$$

The log of a product is the sum of the logs

$$\log_a xy = \log_a x + \log_a y$$

The log of a quotient is the difference of the logs

$$\log_a \frac{x}{y} = \log_a x - \log_a y$$

The exponent on the argument of the log is the coefficient of the log

$$\log_a x^n = n \cdot \log_a x$$

Change of base formula

$$\log_a x = \frac{\log_b x}{\log_b a} = \frac{\log x}{\log a} = \frac{\ln x}{\ln a}$$

Chapter 5 - Partial Fraction Decomposition

Every linear factor in the denominator needs a constant factor in the numerator of the expansion.

$$\frac{3x + 2}{(x - 3)(x + 5)} = \frac{A}{x - 3} + \frac{B}{x + 5}$$

Every irreducible quadratic factor in the denominator needs a linear factor in the numerator of the expansion.

$$\frac{5x^2 - 2x + 7}{(3x + 2)(5x^2 + x + 4)} = \frac{A}{3x + 2} + \frac{Bx + C}{5x^2 + x + 4}$$

Repeated factors (factors with a multiplicity greater than one) need to have a term in the expansion for each possible power. The multiplicity does not change whether the factor is linear or quadratic.

$$\frac{4x + 7}{(x + 2)^3(x^2 + 5)^2} = \frac{A}{x + 2} + \frac{B}{(x + 2)^2} + \frac{C}{(x + 2)^3} + \frac{Dx + E}{x^2 + 5} + \frac{Fx + G}{(x^2 + 5)^2}$$

Two sides of an equation are equal only if the corresponding terms are equal. Consider the basic equation:

$$4x^2 - 3x + 2 = A(x - 3)(x + 5) + B(x + 2)(x + 5) + C(x - 3)(x + 2)$$
$$4x^2 - 3x + 2 = A(x^2 + 2x - 15) + B(x^2 + 7x + 10) + C(x^2 - x - 6)$$

Setting corresponding terms from each side equal gives a system of equations.

$$\begin{array}{rclclcl} x^2: & 4 & = & A & + & B & + & C \\ x: & -3 & = & 2A & + & 7B & - & C \\ 1: & 2 & = & -15A & + & 10B & - & 6C \end{array}$$

Chapter 6 - Matrices

Matrix multiplication is performed by pairing up numbers in a row from the first matrix and a column from the second matrix. These pairs of numbers are multiplied together and then added to find the corresponding number in the matrix product.

If the inner dimensions aren't the same, then we're unable to pair up the numbers and the matrix multiplication is undefined. If the multiplication is defined, then the dimensions of the product are the number of rows from the first matrix by the number of columns of the second matrix.

$$\text{Consider the product } \begin{bmatrix} 1 & -2 & 4 \\ 3 & 1 & -5 \end{bmatrix} \begin{bmatrix} -1 & 2 \\ 3 & 1 \\ 4 & -3 \end{bmatrix} = \begin{bmatrix} 9 & -12 \\ -20 & 22 \end{bmatrix}$$

The elements in the product can be found as follows:

$R_1:$	1	-2	4		$R_1:$	1	-2	4	
$\times C_1:$	-1	3	4		$\times C_2:$	2	1	-3	
<hr/>					<hr/>				
	-1	-6	+16	= 9		2	-2	-12	= -12
$R_2:$	3	1	-5		$R_2:$	3	1	-5	
$\times C_1:$	-1	3	4		$\times C_2:$	2	1	-3	
<hr/>					<hr/>				
	-3	+3	-20	= -20		6	+1	+15	= 22

(Note: The above structure was created using nested matrices. There is a 2×2 matrix with 3×5 matrices within each of the four cells. Don't type this note.)

Chapter 7 - Sequences and Series

Arithmetic Sequences & Series (Linear)

Common Difference: $d = a_{n+1} - a_n$

General Term: $a_n = a_1 + (n - 1)d$

n^{th} partial sum: $S_n = \frac{n}{2}(a_1 + a_n)$

Geometric Sequences & Series (Exponential)

Common Ratio: $r = \frac{a_{n+1}}{a_n}$

General Term: $a_n = a_1 \cdot r^{n-1}$

n^{th} partial sum: $S_n = \frac{a_1(1 - r^n)}{1 - r}$

Infinite sum: $S = \frac{a_1}{1 - r}, |r| < 1$

Permutations and Combinations

$${}_n P_r = \frac{n!}{(n - r)!}$$

$${}_n C_r = \frac{n!}{r!(n - r)!}$$

Binomial Expansion Theorem

$$(x + y)^n = \sum_{k=0}^n \binom{n}{k} x^{n-k} y^k$$

Chapter 8 - Conics

Determining the Type of Conic

Circle	$x^2 + y^2 = 9$
Ellipse	$9x^2 + 4y^2 = 144$
Point	$4x^2 + 9y^2 = 0$
No Graph	$3x^2 + 7y^2 = -4$
Hyperbola	$5x^2 - 4y^2 = 18$
Intersecting Lines	$4x^2 - 3y^2 = 0$
Parabola	$x^2 - 5y + 7 = 0$
Parallel Lines	$x^2 - x - 6 = 0$
Line	$3x + 4y = 12$

Standard Forms

Conic	Horizontal	Vertical
Parabola	$y^2 = 4px$	$x^2 = 4py$
Ellipse	$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$	$\frac{y^2}{a^2} + \frac{x^2}{b^2} = 1$
Hyperbola	$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$	$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$