Exam 2 - Chapter 8 Math 122 Name : _____
70 Points - Take Home Calculus & Analytic Geometry II

1. Find the average water level in feet on the Sangamon River at Illinois Rt 48 in Decatur, IL, for August 25, 2003 through September 23, 2003. (10 points).

Collect the water level data from the US Geological Survey website (URL given below) and record it on the attached sheet.

From the web page, request the "Gauge height" data in either fixed-width table or tab separated output. Although the output includes data at 15 minute intervals, only record the midnight (00:00:00) and noon (12:00:00) readings.

Sample Output

Date and Time (CST)	Stage
08/25/2003 00:00	2.25
08/25/2003 12:00	2.26
08/26/2003 00:00	2.25
08/26/2003 12:00	2.27
08/27/2003 00:00	2.27
08/27/2003 12:00	2.28
08/28/2003 00:00	2.26
08/28/2003 12:00	2.24
08/29/2003 00:00	2.24
08/29/2003 12:00	2.28
08/30/2003 00:00	2.30
08/30/2003 12:00	2.25
08/31/2003 00:00	2.22
08/31/2003 12:00	3.17

Record the midnight readings from 8/25/2003 through 9/24/2003 so that you have a full 30 days of information (but 31 endpoints). Also record the readings at noon from 8/25/2003 to 9/23/2003.

The first week's worth of data is shown in the table above, but you will need to gather the rest of the information from the Internet. You can only request information for the last 31 days, but with the data above, you should be able to get all of the information you need.

http://waterdata.usgs.gov/nwis/uv/?site_no=05573540

a. Using the 31 values recorded at midnight, approximate the average water stage using ...

   i. the left-hand endpoints

   ii. the right-hand endpoints

   iii. the trapezoidal method

   iv. Simpson's method

b. Using the 30 values recorded at noon, approximate the average water stage using the midpoint method.

2. The Laplace transform, $\mathcal{L}\{f(x)\}$, is useful in differential equations. Find the following Laplace transforms. Note that the Laplace Transform is a function of *s* only, there is no *x* in the transform. Look at problems 8.8.56-57 in the book for additional information. The restrictions on *s* that are given in the book are necessary so that the improper integral converges, be sure to state proper restrictions. Most of the integrals will require integration by parts if done by hand, however you may use a table of integrals. An example problem has been worked for you.

$$\mathcal{L}\{f(x)\} = \int_0^{+\infty} e^{-sx} f(x)dx$$

Work any five (5) of the following transforms. (10 points)

a. $\mathcal{L}\{a\}$

b. $\mathcal{L}\{ax\}$

c. $\mathcal{L}\{\sin ax\}$

d. $\mathcal{L}\{\cos ax\}$

e. $\mathcal{L}\{\sinh ax\}$

f. $\mathcal{L}\{\cosh ax\}$

g. $\mathcal{L}\{xe^{-ax}\}$

Example: Find $\mathcal{L}\{e^{ax}\}$

$$= \int_0^{+\infty} e^{-sx}e^{ax}dx = \int_0^{+\infty} e^{-(s-a)x}dx$$

let $u = -(s-a)x$, $du = -(s-a)dx$

$u\big|_{x\to+\infty} = -\infty$ *if* $s > a$, $u\big|_{x=0} = 0$

$$= \frac{-1}{s-a}\int_0^{-\infty} e^u du = \frac{1}{s-a}\int_{-\infty}^{0} e^u du$$

$$= \frac{e^u}{s-a}\Big|_{-\infty}^{0} = \frac{1}{s-a}\left(e^0 - \lim_{b\to-\infty} e^b\right)$$

$$= \frac{1-0}{s-a} = \frac{1}{s-a},\ s > a$$

3. Use a table of integrals to find the integral. In each case, copy the number of the formula and the integration formula itself. Then give the values of any variables (ex: *a* or *u*). Finally, find the integral and simplify. (10 points)

a. Problem 8.6.10

b. Problem 8.6.20

c. Problem 8.6.24

d. Problem 8.6.30

e. Problem 8.6.38

Example solution to 8.6.9

$$\int \frac{1}{x\sqrt{3x-4}}\,dx$$

The matching formula is #108

$$\int \frac{du}{u\sqrt{a+bu}} = \frac{2}{\sqrt{-a}}\tan^{-1}\sqrt{\frac{a+bu}{-a}} + C$$

with $a = -4, u = 3x, du = 3dx$.

Adjust constants in the original integral to match the formula.

$$\int \frac{3}{3x\sqrt{3x-4}}\,dx$$

Use the formula

$$\frac{2}{\sqrt{-(-4)}}\tan^{-1}\sqrt{\frac{3x-4}{-(-4)}} + C$$

and simplify

$$\tan^{-1}\left(\frac{\sqrt{3x-4}}{2}\right) + C$$

4. The Gamma function converges only for positive values of x and is defined by $\Gamma(x) = \int_0^{+\infty} t^{x-1} e^{-t}\, dt$. (10 points)

a. Find $\Gamma(1) = \int_0^{+\infty} t^0 e^{-t}\, dt$

b. Find $\Gamma(6)$ by using tabular integration by parts.

c. Use integration by parts to prove that $\Gamma(x+1) = x\Gamma(x), \forall x > 0$

d. When n is a positive integer, $\Gamma(n)$ can be written in terms of another mathematical function. Write the equivalent expression for $\Gamma(n)$.

e. Use the fact that $\Gamma(0.5) = \sqrt{\pi}$ to find $\Gamma(4.5)$.

5. Use a computer algebra system (Derive, Maple, Mathematica, TI-89/92) to find the following integrals. Use the equation editor to answer this question. Type the original problem and the solution in the form "problem = solution". (10 points)

a. Problem 8.6.42

b. Problem 8.6.50

c. Problem 8.6.56

d. Problem 8.6.62

e. Problem 8.6.70

6. Work ten (10) of the following problems by hand. There must be exactly two problems from each section (8.2, 8.3, 8.4, 8.5, and 8.8). Show all work. Clearly identify the section number and problem. Attach the problems in section order. You may use the reduction formulas where necessary, but otherwise do not use the table of integrals. You may use a CAS to check your answer, but show work. (20 points)

a. Problem 8.2.8

b. Problem 8.2.20

c. Problem 8.2.38

d. Problem 8.3.8

e. Problem 8.3.30

f. Problem 8.3.44

g. Problem 8.4.12

h. Problem 8.4.18

i. Problem 8.4.40

j. Problem 8.5.14

k. Problem 8.5.20

l. Problem 8.5.26

m. Problem 8.8.8

n. Problem 8.8.22

o. Problem 8.8.42

Water Level on Sangamon River at Decatur, IL, for Aug/Sep 2003.

#	Date	Midnight Reading	Noon Reading		#	Date	Midnight Reading	Noon Reading
1	25-Aug	2.25	2.26		17	10-Sep		
2	26-Aug	2.25	2.27		18	11-Sep		
3	27-Aug	2.27	2.28		19	12-Sep		
4	28-Aug	2.26	2.24		20	13-Sep		
5	29-Aug	2.24	2.28		21	14-Sep		
6	30-Aug	2.30	2.25		22	15-Sep		
7	31-Aug	2.22	3.17		23	16-Sep		
8	01-Sep				24	17-Sep		
9	02-Sep				25	18-Sep		
10	03-Sep				26	19-Sep		
11	04-Sep				27	20-Sep		
12	05-Sep				28	21-Sep		
13	06-Sep				29	22-Sep		
14	07-Sep				30	23-Sep		
15	08-Sep				31	24-Sep		
16	09-Sep							