Take Home Exam - Chapter 9 Math 122 Name : ________________________

30 Points Calculus & Analytic Geometry II

1. **Sick Horse** (6 pts)

Newton's Law of Cooling (or Warming) states that the rate of change of the temperature of a body is proportional to the difference between the body's temperature and the surrounding medium's temperature. (6 pts)

a. Let y represent the temperature of the body and m represent the temperature of the medium at time t. Write a differential equation that will model Newton's Law of Cooling. Note, both y and m are functions of t. (1 pt)

b. Solve the differential equation in part (a) for y. (2 pts)

c. A veterinarian wants to find the temperature of a sick horse. The readings on the thermometer follow Newton's Law. At the time of insertion, the thermometer reads 82°F. After three minutes, the reading is 90°F, and is 94° three minutes after that. A sudden convulsion destroys the thermometer before a final reading can be obtained. What is the horse's temperature? Assume that the temperature of the surrounding medium doesn't change over the six minute period. (2 pts)

d. How long would it have taken to measure the horse's temperature if the thermometer hadn't been destroyed? Hint: find the time it would take to be within ±0.5°F of the actual temperature. (1 pt)

2. **Height of a whiffle ball** (13 pts)

Newton's Second Law (restricted to vertical motion) states that the product of the constant mass and the acceleration of a body moving vertically is the sum of the external forces acting vertically on the body.

$$\begin{aligned} my'' &= -mg - ky' \\ y(0) &= y_0 \\ y'(0) &= v_0 \end{aligned}$$

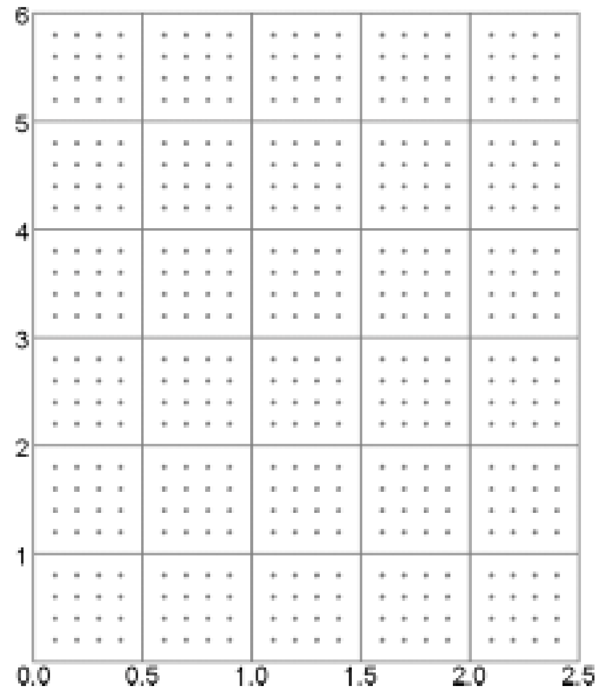
a. Since velocity is the derivative of position $(v = y')$, rewrite the viscous damping model as an initial value first order differential equation in v instead of a second order differential equation in y. (1 pt)

b. Solve the differential equation using the method of integrating factors to find $v(t)$. (2 pts)

c. Find the limiting velocity of the whiffle ball, $v_\infty = \lim_{t \to +\infty} v(t)$. (1 pt)

d. Let g=9.8 m/s^2, v_0=15 m/s, k/m = 3/s, and y_0=1.75 m. Use Euler's method with h=0.1 s to make graph the height of the whiffle ball over the time interval $0 < t < 2.4$ s. (4 pts)

n	t_n	y_n
0	0.0	1.75
1	0.1	
2	0.2	
3	0.3	
4	0.4	
5	0.5	
6	0.6	
7	0.7	
8	0.8	
9	0.9	
10	1.0	
11	1.1	
12	1.2	
13	1.3	
14	1.4	
15	1.5	
16	1.6	
17	1.7	
18	1.8	
19	1.9	
20	2.0	
21	2.1	
22	2.2	
23	2.3	
24	2.4	



e. Integrate the velocity function to find the position function, $y(t)$. Don't forget about the initial value for y. (2 points)

f. Let g=9.8 m/s^2, v_0=15 m/s, k/m=3 s^{-1}, and y_0=1.75 m. Graph the position function on your calculator (you don't need to copy it here) and answer the following questions based on the graph? (3 points)

i. How long before the whiffle ball hits the ground?

ii. What is the maximum height of the whiffle ball?

iii. How long does the whiffle ball take to reach its maximum height?

3. **Cold Medicine Effectiveness** (11 pts)

Most cold pills are designed to dissolve quickly when taken and then release their medications in the GI tract at a constant rate over half an hour. The level of antihistamine in the GI tract, $x(t)$, changes at a rate that is proportional to the amount in the GI tract. The antihistamine that leaves the GI tract enters the blood system. For purposes of this problem, the "blood" includes the blood and the tissues where the medication does it work.

The level of antihistamine in the blood, $y(t)$, builds up from zero but then falls as the kidneys and liver do their job of clearing foreign substances from the blood. The rate at which the antihistamine is removed is proportional to the amount of antihistamine present in the blood. The Balance Law states that the net rate of change is equal to the rate in minus the rate out.

If A is the units of antihistamine taken at time $t = 0$ hrs, the rate at which the antihistamine level in the GI tract changes is

$$\frac{dx}{dt} = \begin{cases} 2A - k_1 x, & 0 \le t \le 0.5, \quad x(0) = 0 \\ -k_1 x, & t > 0.5, \quad \text{See note} \end{cases}$$

Note: The initial value of antihistamine in the GI tract for the second piece of the function is equal to $x(0.5)$, found from the first piece of the function. The $2A$ is because it takes 1/2 hr for A mg to enter the bloodstream, so the hourly rate is $2A$.

The antihistamine leaves the GI tract at the same rate at which it enters the blood system, so the rate at which the antihistamine level in the blood changes is

$$\frac{dy}{dt} = k_1 x - k_2 y, \;\; t \ge 0, \;\; y(0) = 0$$

One pharmaceutical company estimates that the rate constants for the antihistamine in the cold pills it makes are:

$$k_1 = 0.6931/hr \text{ and } k_2 = 0.0231/hr$$

The k_2 value (clearance coefficient) is for an average person. It is often much lower for the old and the sick than the young and healthy.

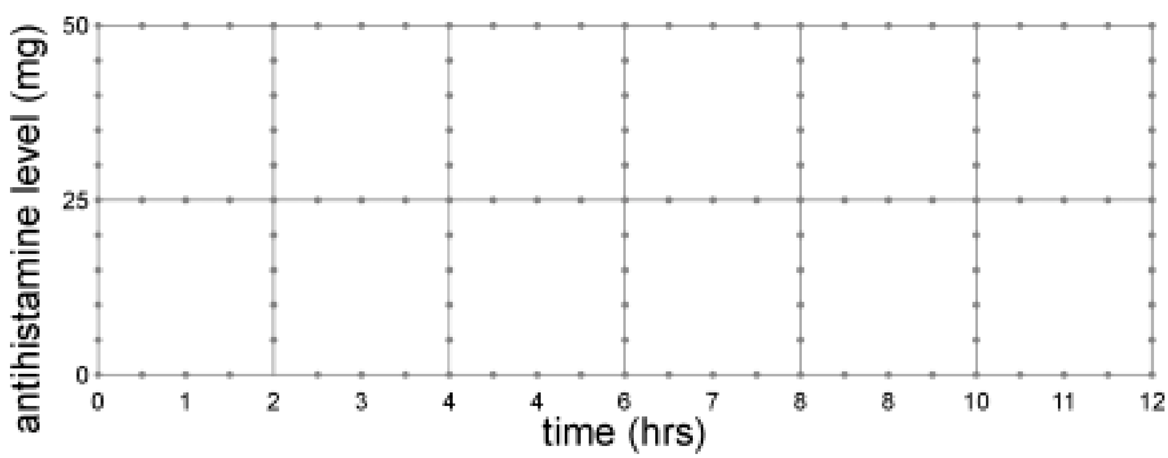
A person takes two cold pills, each containing 25 mg of diphenydramine HCl (antihistamine), so $A = 50$ mg.

Show your work to parts (a) and (b) on separate sheets!

a. Find the amount of antihistamine in the GI tract as a function of t. Give your answer in piecewise form. (4 pts)

b. Find the amount of antihistamine in the blood as a function of t. Give your answer in piecewise form. (4 pts)

c. Graph both $x(t)$ and $y(t)$ versus t on the same coordinate system over the time period 0 hrs $\leq t \leq$ 12 hrs. (2 pts)



d. How much antihistamine remains in the blood system 24 hours after taking the pills? (1 pt)