Mathematical Notation

Math 122 - Calculus & Analytic Geometry II

Name : ____________________________

Use Word or WordPerfect to recreate the following documents. Each article is worth 10 points and can be printed and given to the instructor or emailed to the instructor at james@richland.edu.   If you use Microsoft Works to create the documents, then you must print it out and give it to the instructor as he can't open those files.

Type your name at the top of each document.

For expressions or equations, you should use the equation editor in Word or WordPerfect. The documents were created using a 14 pt Times New Roman font with standard 1" margins.

For individual symbols (μ, σ, etc), you can insert symbols. In Word, use "Insert / Symbol" and choose the Symbol font. For WordPerfect, use Ctrl-W and choose the Greek set.

The due date for each of these documents is the day after the exam for that chapter. While the material is not due until after the exam, it is recommended that you create it ahead of time because the material will help you review for the exam.

Chapter 7 - Inverse Functions

$$\frac{d}{dx}\left[f^{-1}(x)\right]=\frac{1}{f'\left[f^{-1}(x)\right]} \qquad \frac{dy}{dx}=\frac{1}{dx/dy}$$

$$e=\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^{x}=\lim_{x\to 0}(1+x)^{1/x}$$

$$\frac{d}{dx}\left[e^{x}\right]=e^{x} \quad \frac{d}{dx}\left[a^{x}\right]=a^{x}\cdot\ln a \quad \frac{d}{dx}[\ln x]=\frac{1}{x} \quad \frac{d}{dx}[\log_a x]=\frac{1}{x\cdot\ln a}$$

$$\frac{d}{dx}\left[\sin^{-1}x\right]=\frac{1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}\left[\cos^{-1}x\right]=\frac{-1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}\left[\tan^{-1}x\right]=\frac{1}{1+x^2} \qquad \frac{d}{dx}\left[\cot^{-1}x\right]=\frac{-1}{1+x^2}$$

$$\frac{d}{dx}\left[\sec^{-1}x\right]=\frac{1}{|x|\sqrt{x^2-1}} \qquad \frac{d}{dx}\left[\csc^{-1}x\right]=\frac{-1}{|x|\sqrt{x^2-1}}$$

If *f* and *g* are differentiable and $\lim_{x\to a} f(x)$ and $\lim_{x\to a} g(x)$ are both 0 or ∞, then

$$\lim_{x\to a}\left(\frac{f(x)}{g(x)}\right)=\lim_{x\to a}\left(\frac{f'(x)}{g'(x)}\right)$$

$$\frac{d}{dx}[\sinh x]=\cosh x \qquad \frac{d}{dx}[\cosh x]=\sinh x$$

$$\frac{d}{dx}[\tanh x]=\operatorname{sech}^2 x \qquad \frac{d}{dx}[\coth x]=-\operatorname{csch}^2 x$$

$$\frac{d}{dx}[\operatorname{sech} x]=-\operatorname{sech} x\tanh x \qquad \frac{d}{dx}[\operatorname{csch} x]=-\operatorname{csch} x\coth x$$

Chapter 8 - Techniques of Integration

Integration by Parts

$$\int u\,dv = uv - \int v\,du$$

Reduction Formulas

$$\int \sin^n x\,dx = -\frac{1}{n}\sin^{n-1} x\cos x + \frac{n-1}{n}\int \sin^{n-2} x\,dx$$

$$\int \cos^n x\,dx = \frac{1}{n}\cos^{n-1} x\sin x + \frac{n-1}{n}\int \cos^{n-2} x\,dx$$

$$\int \tan^n x\,dx = \frac{1}{n-1}\tan^{n-1} x - \int \tan^{n-2} x\,dx$$

$$\int \sec^n x\,dx = \frac{1}{n-1}\sec^{n-2} x\tan x + \frac{n-2}{n-1}\int \sec^{n-2} x\,dx$$

Trigonometric Substitution

For $\sqrt{a^2 - x^2}$, use $x = a\sin\theta$

For $\sqrt{a^2 + x^2}$, use $x = a\tan\theta$

For $\sqrt{x^2 - a^2}$, use $x = a\sec\theta$

Special Trigonometric Substitution

$$\sin x = \frac{2u}{1+u^2} \quad \cos x = \frac{1-u^2}{1+u^2} \quad dx = \frac{2}{1+u^2}du$$

Errors with Numerical Integration

$$\left|E_M\right| \le \frac{(b-a)^3 K_2}{24n^2} \quad \left|E_T\right| \le \frac{(b-a)^3 K_2}{12n^2} \quad \left|E_S\right| \le \frac{(b-a)^5 K_4}{180n^4}$$

Chapter 9 - Differential Equations

First order, linear differential equations can be written in the form
$\frac{dy}{dx} + p(x)y = q(x)$ or $y' + p(x)y = q(x)$.

The integrating factor for a first-order, linear differential equation is $\mu = e^{\int p(x)dx}$. After finding the integrating factor, multiply both sides of the equation by it to get $\mu y' + \mu p(x)y = \mu q(x)$. The left side of the equation is the derivative of a product and so the equation can be written as $\frac{d}{dx}[\mu y] = \mu q(x)$. This equation is separable and can then be integrated to find the solution.

Models where the rate of growth (decay) is proportional to the amount present can be written as $\frac{dy}{dt} = ky$ and give the model $y = Ce^{kt}$.

Models where the rate of growth is proportional to the amount present and the amount not present can be written as $\frac{dy}{dt} = ky(L - y)$ and give the model
$y = \frac{L}{1 + Ce^{-kt}}$.

The solution to a second order, homogenous linear differential equation $y'' + py' + q = 0$ can be found by finding the roots m_1 and m_2 to the equation $m^2 + pm + q = 0$ and then using the form $y(x) = c_1 e^{m_1 x} + c_2 e^{m_2 x}$, $y(x) = c_1 e^{mx} + c_2 x e^{mx}$, or $y(x) = e^{ax}(c_1 \cos bx + c_2 \sin bx)$.

Chapter 10 - Infinite Series

Maclaurin polynomial

$$p_n(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 + \cdots + \frac{f^{(n)}(0)}{n!}x^n$$

$$p_n(x) = \sum_{k=0}^{n} \frac{f^{(k)}(0)}{k!}x^k$$

Taylor Polynomial about $x = a$

$$p_n(x) = \sum_{k=0}^{n} \frac{f^{(k)}(a)}{k!}(x-a)^k$$

$\{a_n\}$ is increasing if $a_{n+1} - a_n \geq 0$, $a_{n+1}/a_n \geq 1$, or $f'(x) \geq 0$ and decreasing if $a_{n+1} - a_n \leq 0$, $a_{n+1}/a_n \leq 1$, or $f'(x) \leq 0$.

The sum of an infinite geometric series is $\sum_{k=0}^{\infty} ar^k = \frac{a}{1-r}, \ |r| < 1$

Every power series will either converge at a point, converge for all real numbers, or converge on a finite open interval.

Common Maclaurin series

$$e^x = \sum_{k=0}^{\infty} \frac{x^k}{k!} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$$

$$\cos x = \sum_{k=0}^{\infty} (-1)^k \frac{x^{2k}}{(2k)!} = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots$$

$$(1+x)^m = 1 + \sum_{k=1}^{\infty} \frac{m(m-1)\cdots(m-k+1)}{k!}x^k$$

Chapter 11 - Analytic Geometry

$$x = r\cos\theta \quad y = r\sin\theta \quad r^2 = x^2 + y^2 \quad \tan\theta = \frac{y}{x}$$

$$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{r\cos\theta + \sin\theta \dfrac{dr}{d\theta}}{-r\sin\theta + \cos\theta \dfrac{dr}{d\theta}}$$

$$L = \int_{\alpha}^{\beta} \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta \qquad A = \int_{\alpha}^{\beta} \frac{1}{2} r^2 d\theta$$

Standard Forms for Conic Sections

Conic	Horizontal	Vertical
Parabola	$y^2 = 4px$	$x^2 = 4py$
Ellipse	$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$	$\frac{y^2}{a^2} + \frac{x^2}{b^2} = 1$
Hyperbola	$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$	$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$

Eliminating the xy-term

Choose an angle θ such that $\cot 2\theta = \frac{A - C}{B},\ 0° < 2\theta < 180°$

Polar Equation of Conics

Relation of Directrix to Pole

Right	Left	Above	Below
$r = \frac{ed}{1 + e\cos\theta}$	$r = \frac{ed}{1 - e\cos\theta}$	$r = \frac{ed}{1 + e\sin\theta}$	$r = \frac{ed}{1 - e\sin\theta}$

e = eccentricity (Ellipse if $0 < e < 1$, Parabola if $e = 1$, Hyperbola if $e > 1$)
d = distance from pole to directrix.