

Materials Needed:

Dice

Instructions:

Two random variables will be studied. These are x , the number that is obtained when a single die is rolled, and \bar{x} , the average value that is obtained when a single die is rolled 40 times.

1. Do you expect to get the same average each time you roll 40 dice? Why or why not?

2. Roll the die 40 times and record the rolled values in the table.

Record the 40 rolls here

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3. Average the results of the 40 rolls. This is one observation of the random variable \bar{x} .

Record your average here:

4. Roll the die another 40 times and obtain the value of \bar{x} for this sample.

Record the 40 rolls here

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Record your average here:

5. Collect the \bar{x} values from each person in the class and record in the table.

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6. Enter the sample means into the computer. Find the mean, variance, and standard deviation and record the information in the bottom row of question 13.
7. Generate a histogram. Copy the histogram below. Does it have the general shape indicated by the Central Limit Theorem?
8. Use the computer to generate a normal probability plot (do not copy the plot here). Does the data appear normal? How can you tell?

9. Complete this table for the theoretical distribution of a single roll of the die.

| x | 1 | 2 | 3 | 4 | 5 | 6 | Total |
|----------------|---------------|---------------|---------------|----------------|----------------|---------------|-------|
| $p(x)$ | $\frac{1}{6}$ | | $\frac{1}{6}$ | | $\frac{1}{6}$ | | 1 |
| $x \cdot p(x)$ | $\frac{1}{6}$ | | $\frac{3}{6}$ | $\frac{4}{6}$ | | $\frac{6}{6}$ | |
| $x^2 p(x)$ | $\frac{1}{6}$ | $\frac{4}{6}$ | | $\frac{16}{6}$ | $\frac{25}{6}$ | | |

10. Use the formulas from section 4.2 (given below) to find the mean, variance, and standard deviation for the random variable x . Record your answers in the first row of the table in question 13.

The mean is $\mu = \sum x \cdot p(x)$.

The variance is $\sigma^2 = \left(\sum x^2 p(x) \right) - \mu^2$.

The standard deviation is $\sigma = \sqrt{\sigma^2}$

11. Copy the formulas from the Central Limit Theorem.

a. The theoretical mean of the sample means

$$\text{Mean}(\bar{x}) = \mu_{\bar{x}} =$$

b. The theoretical variance of the sample means

$$\text{Var}(\bar{x}) = \sigma_{\bar{x}}^2 =$$

c. The theoretical standard deviation of the sample means

$$\text{SD}(\bar{x}) = \sigma_{\bar{x}} =$$

12. Use the formulas from question 11 with the theoretical values for the population x that were found in question 10 to find the theoretical values for the sample mean, \bar{x} . Record your answers in the middle row of question 13.
13. Record the theoretical values for outcome of the roll of a single die (found in question 10), the theoretical values for the mean of forty rolls of the die (found in question 12), and the observed values for our sample means (found in question 6).

| | Mean | Variance | Standard Deviation |
|---|-------------------|------------------------|----------------------|
| Theoretical Values for x . | $\mu =$ | $\sigma^2 =$ | $\sigma =$ |
| Theoretical Values for \bar{x} , $n=40$ | $\mu_{\bar{x}} =$ | $\sigma_{\bar{x}}^2 =$ | $\sigma_{\bar{x}} =$ |
| Observed Values for \bar{x} , $n=40$ | $\bar{x} =$ | $s^2 =$ | $s =$ |

14. Comment on the closeness of the observed values to what the Central Limit Theorem predicted the values should be.