Mathematical Notation

Math 190 - Calculus for Business and Social Science

Use Word or WordPerfect to recreate the following documents. Each article is worth 10 points and should be emailed to the instructor at james@richland.edu. If you use Microsoft Works to create the documents, then you must print it out and give it to the instructor as he can't open those files.

Type your name at the top of each document.

Include the title as part of what you type. The lines around the title aren't that important, but if you will type ----- at the beginning of a line and hit enter, both Word and WordPerfect will draw a line across the page for you.

For expressions or equations, you should use the equation editor in Word or WordPerfect. The instructor used WordPerfect and a 14 pt Times New Roman font with 0.75" margins, so they may not look exactly the same as your document. The equations were created using 14 pt font.

For individual symbols (μ, σ, etc) within the text of a sentence, you can insert symbols. In Word, use "Insert / Symbol" and choose the Symbol font. For WordPerfect, use Ctrl-W and choose the Greek set. However, it's often easier to just use the equation editor as expressions are usually more complex than just a single symbol. If there is an equation, put both sides of the equation into the same equation editor box instead of creating two objects.

There are instructions on how to use the equation editor in a separate document or on the website. Be sure to read through the help it provides. There are some examples at the end that walk students through the more difficult problems. You will want to read the handout on using the equation editor if you have not used this software before.

If you fail to type your name on the page, you will lose 1 point. Don't type the grayed out hints or reminders at the bottom of each page.

These notations are due at the beginning of class on the day of the exam for that chapter. That is, the chapter 1 notation is due on the day of the chapter 1 test. Late work will be accepted but will lose 20% of its value per class period.

Chapter 1 - Preliminaries

Absolute Value $|x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$ use a 2×2 matrix for the rhs

Rational Exponents $x^{m/n} = \sqrt[n]{x^m}$

Difference of two squares $x^2 - y^2 = (x-y)(x+y)$

Perfect square trinomial $x^2 \pm 2xy + y^2 = (x \pm y)^2$

Sum/Difference of two cubes $x^3 \pm y^3 = (x \pm y)(x^2 \mp xy + y^2)$

Example of factoring with rational exponents

$$\frac{2}{3}(5)\left(2x^2+3\right)^{3/2}(5x-2)^{-1/3} + \frac{3}{2}(4x)\left(2x^2+3\right)^{1/2}(5x-2)^{2/3}$$

$$= \frac{10}{3}\left(2x^2+3\right)^{3/2}(5x-2)^{-1/3} + 6x\left(2x^2+3\right)^{1/2}(5x-2)^{2/3}$$

$$= \frac{2}{3}\left(2x^2+3\right)^{1/2}(5x-2)^{-1/3}\left[5\left(2x^2+3\right)+9x(5x-2)\right]$$

$$= \frac{2}{3}\left(2x^2+3\right)^{1/2}(5x-2)^{-1/3}\left(10x^2+15+45x^2-18x\right)$$

$$= \frac{2}{3}\left(2x^2+3\right)^{1/2}(5x-2)^{-1/3}\left(55x^2-18x+15\right)$$

Quadratic Formula $x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$

Distance Formula $d = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$

Slope $m = \dfrac{rise}{run} = \dfrac{\Delta y}{\Delta x} = \dfrac{y_2-y_1}{x_2-x_1}$

Point Slope form of line $y - y_1 = m(x - x_1)$

Chapter 2 - Functions, Limits, and the Derivative

Composition of functions $(f \circ g)(x) = f[g(x)]$

Limits are what separate calculus from algebra. A limit is not concerned with what happens at a particular value, only what happens as you approach that value.

Limits can usually be evaluated by substituting the value into the expression unless that causes a problem like division by zero. If there is a problem, then we try to manipulate the expression to eliminate the problem.

Example of an indeterminate form. The original form is 0/0 and so we rationalize the numerator by multiplying by 1 in the form of the conjugate over itself and simplify. Notice that we continue to write the limit in front until that point where we actually plug in the value of 2.

$$\lim_{x\to 2}\frac{2-\sqrt{x+2}}{x-2} = \lim_{x\to 2}\left(\frac{2-\sqrt{x+2}}{x-2}\right)\left(\frac{2+\sqrt{x+2}}{2+\sqrt{x+2}}\right)$$

$$= \lim_{x\to 2}\frac{4-(x+2)}{(x-2)\left(2+\sqrt{x+2}\right)} = \lim_{x\to 2}\frac{2-x}{(x-2)\left(2+\sqrt{x+2}\right)}$$

$$= \lim_{x\to 2}\frac{-1}{2+\sqrt{x+2}} = \frac{-1}{2+\sqrt{2+2}} = \frac{-1}{2+\sqrt{4}} = -\frac{1}{4}$$

A function is continuous at a point if the function is defined at that point, the limit as you approach that point exists, and the value of the function is equal to the limit.

Average rate of change $m_{\sec} = \dfrac{f(x+h)-f(x)}{h}$

Instantaneous rate of change $m_{\tan} = \lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$

Chapter 3 - Differentiation

Power Rule $\frac{d}{dx}\left[x^n\right] = nx^{n-1}$

Product Rule $(fg)' = fg' + f'g$

Quotient Rule $\left(\frac{f}{g}\right)' = \frac{gf' - fg'}{g^2}$

Chain Rule $\frac{d}{dx}\left[f\left[g(x)\right]\right] = f'\left[g(x)\right]g'(x)$

Chain Rule $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$

In English, the Chain Rule says to take the derivative of the outside function times the derivative of the inside function.

In economics, "marginal" means derivative. So marginal cost is C', marginal revenue is R', and marginal profit is P'.

Average Cost $\overline{C}(x) = \frac{C(x)}{x}$

Elasticity of Demand $E(p) = -\frac{pf'(p)}{f(p)}$

Demand is elastic if elasticity is greater than 1, unitary if elasticity is 1, and inelastic if elasticity is less than 1.

A differential is a change in a variable. A derivative is a rate of change and is the ratio of two differentials. So *dx* and *dy* are differentials and *dy*/*dx* is a derivative. $dx = \Delta x$ and $dy = f'(x)\Delta x \approx \Delta y$.

Local Linear Approximation $f(x) = f(x_0) + \Delta y \approx f(x_0) + f'(x_0)\Delta x$

Chapter 4 - Applications of the Derivative

A function is increasing where the first derivative is positive and decreasing where the first derivative is negative.

A critical point is a value in the domain of the function where the first derivative is either zero or undefined.

A function is concave up when the second derivative is positive and concave down where the second derivative is negative.

An inflection point is a point where the concavity changes.

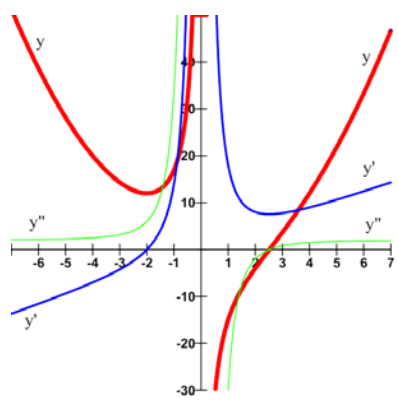
A continuous function has a relative maximum at a critical point if the function is increasing to the left and decreasing to the right of that point. A continuous function has a relative minimum at a critical point if the function is decreasing to the left and increasing to the right of that point.

A continuous function has a relative maximum at a critical point if the second derivative is negative at that point and a relative minimum at a critical point if the second derivative is positive at that point.

Vertical asymptotes of a rational function occur when the denominator is zero. Horizontal asymptotes are found by taking the limit as $x \to \pm\infty$.

A continuous function on a closed interval will have both an absolute maximum and an absolute minimum.

The graph of the function $y = x^2 - 16/x$ is shown to the right. Notice that where y has a relative minimum, y' crosses the x-axis and where there is an inflection point, y'' crosses the x-axis.



Use winplot to create the graph

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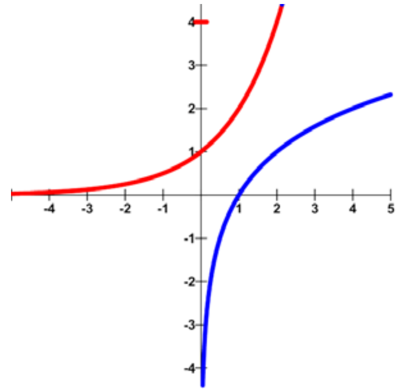
Chapter 5 -Exponential and Logarithmic Functions



The graph of $y = 2^x$ and $y = \log_2 x$ on the same graph.

Notice they are reflections of each other about the line $y = x$ (inverses of each other). The exponential function has a domain of all real numbers and a range of $y > 0$ while the logarithmic function has a domain of $x > 0$ and a range of all real numbers.

Use winplot to create graph

Definition of *e* $\quad e = \lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x = \lim_{x\to 0}(1+x)^{1/x}$

Conversion between forms $\quad y = \log_b x \Leftrightarrow x = b^y$

Log of a product $\quad \log_b xy = \log_b x + \log_b y$

Log of a quotient $\quad \log_b \frac{x}{y} = \log_b x - \log_b y$

Log with a power $\quad \log_b x^n = n\log_b x$

Compound Interest $\quad A = P\left(1+\frac{r}{m}\right)^{mt}$

Effective Rate of Interest $\quad r_{eff} = \left(1+\frac{r}{m}\right)^m - 1$

Derivative of exponential $\quad \frac{d}{dx}\left[e^x\right] = e^x$

Derivative of logarithm $\quad \frac{d}{dx}\left[\ln|x|\right] = \frac{1}{x}$

Don't forget to put your name at the top of the page

Chapter 6 - Integration

Since the derivative of a constant is 0, any evidence of the constant that existed in the original function is lost when the derivative is found. When finding the antideriviative, we compensate for this by adding a constant. This shows up as a +*C* at the end of every indefinite integral. The value of *C* can be found for initial value problems.

Power Rule for Integrals $\int x^n dx = \frac{1}{n+1} x^{n+1} + C$

Integral of exponential $\int e^x dx = e^x + C$

Integral of reciprocal $\int \frac{1}{x} dx = \ln|x| dx + C$

Integration by substitution is the antiderivative form of the chain rule.

The definite integral is equal to the area under a curve. This is found by taking a limit of a sum as the sub-intervals get smaller and smaller.

Definite Integral $\int_a^b f(x) dx = \lim_{n \to \infty} \left(\sum_{k=1}^{n} f(x_k) \Delta x \right)$

That definition makes sense, but it's a pain to work with. More useful is the Fundamental Theorem of Calculus, which states that if *F* is any antiderivative of a continuous function *f*, then $\int_a^b f(x) dx = F(b) - F(a)$.

Average Value $f_{ave} = \frac{1}{b-a} \int_a^b f(x) dx$

Consumer's Surplus $CS = \int_0^{\overline{x}} D(x) dx - \overline{p}\,\overline{x}$

Producer's Surplus $PS = \overline{p}\,\overline{x} - \int_0^{\overline{x}} S(x) dx$

Coefficient of Inequality $L = 2\int_0^1 [x - f(x)] dx$

Chapter 7 - Additional Topics in Integration

Integration by parts is based is developed from the product rule for derivatives. $\int u\,dv = uv - \int v\,du$. You need to divide the original integral into two parts, *u* and *dv*. You **must** be able to integrate *dv* and *du* **should** be simpler than *u*. An acronym to help you choose *u* is LAE, which stands for Logarithm functions, Algebraic functions, and Exponential functions where you let *u* be the first kind you encounter. This doesn't always work, but it will work much of the time.

A table of integrals provides a quick way to evaluate some integrals. You may need to make substitutions before looking up the value in the table. A short table of integrals is found in section 7.2 of your text.

Sometimes definite integrals just can't be evaluated algebraically. In these cases, we have numerical methods like the trapezoid rule or Simpson's rule that can be used to approximate the definite integral.

Trapezoid Rule. The *y* value of each interior point is counted twice.

$$\int_a^b f(x)\,dx = \frac{b-a}{2n}\left[y_0 + 2y_1 + 2y_2 + \cdots + 2y_{n-2} + 2y_{n-1} + y_n\right]$$

Simpson's Rule, number of intervals must be even. The *y* values of each interior point alternate between being counted four times and being counted twice.

$$\int_a^b f(x)\,dx = \frac{b-a}{3n}\left[y_0 + 4y_1 + 2y_2 + \cdots + 2y_{n-2} + 4y_{n-1} + y_n\right]$$

Chapter 8 - Calculus of Several Variables

To graph a function of two variables, $z = f(x, y)$, we can either graph a three-dimensional surface or a two-dimensional contour map made up of level curves. In a contour plot, the closer the level curves are together, the more rapidly the value of the function is changing.

A partial derivative is a rate of change with respect to one of the variables while the other variables are held constant. $\frac{\partial f}{\partial x} = f_x$ is the first partial derivative of *f* with respect to *x* and $\frac{\partial f}{\partial y} = f_y$ is the first partial derivative of *f* with respect to *y*.

A critical point is a point in the domain where either both first partial derivatives are zero or at least one of the first partial derivatives does not exist.

Assuming a critical point is the kind where both first partial derivatives are zero, then you can determine whether the value is a maximum or minimum by using the second derivative test.

Second Derivative Test $D = f_{xx} f_{yy} - f_{xy} f_{yx} = f_{xx} f_{yy} - f_{xy}^2$
If $D > 0$ and $f_{xx} < 0$, then the critical point is a relative maximum.
If $D > 0$ and $f_{xx} > 0$, then the critical point is a relative minimum.
If $D < 0$, then the critical point is a saddle point and if $D = 0$, the test is inconclusive.

Lagrange Multipliers.
To find the relative extrema of a function *f* subject to a constraint $g(x, y) = 0$, form a auxiliary function $F(x, y, \lambda) = f(x, y) + \lambda g(x, y)$. Then solve the system of equations that results by setting each partial derivative equal to zero. That is, simultaneously set $F_x = 0$, $F_y = 0$, and $F_\lambda = 0$.