Mathematical Notation

Math 221 - Calculus & Analytic Geometry III

Use Word or WordPerfect to recreate the following documents. Each article is worth 10 points and should be emailed to the instructor at james@richland.edu. If you use Microsoft Works to create the documents, then you must print it out and give it to the instructor as he can't open those files.

Type your name at the top of each document.

Include the title as part of what you type. The lines around the title aren't that important, but if you will type ----- at the beginning of a line and hit enter, both Word and WordPerfect will draw a line across the page for you.

For expressions or equations, you should use the equation editor in Word or WordPerfect. The instructor used WordPerfect and a 14 pt Times New Roman font with 0.75" margins, so they may not look exactly the same as your document. The equations were created using 14 pt font.

For individual symbols (θ, Φ, etc) within the text of a sentence, you can insert symbols. In Word, use "Insert / Symbol" and choose the Symbol font. For WordPerfect, use Ctrl-W and choose the Greek set. However, it's often easier to just use the equation editor as expressions are usually more complex than just a single symbol. If there is an equation, put both sides of the equation into the same equation editor box instead of creating two objects.

There are instructions on how to use the equation editor in a separate document or on the website. Be sure to read through the help it provides. There are some examples at the end that walk students through the more difficult problems. You will want to read the handout on using the equation editor if you have not used this software before.

If you fail to type your name on the page, you will lose 1 point. Don't type the grayed out hints or reminders at the bottom of each page.

These notations are due at the beginning of class on the day of the exam for that chapter. That is, the chapter 12 notation is due on the day of the chapter 12 test. Late work will be accepted but will lose 20% of its value per class period.

Chapter 12 - Vectors

Press Ctrl-B before each vector to make it bold

Dot Products

The dot product is a scalar and is defined as $\mathbf{u} \cdot \mathbf{v} = u_1 v_1 + u_2 v_2 + u_3 v_3$

The dot product is 0 if and only if the vectors are orthogonal.

$\mathbf{u} \cdot \mathbf{v} = \|\mathbf{u}\| \|\mathbf{v}\| \cos\theta$

The orthogonal projection of **v** onto **b** is $\operatorname{proj}_{\mathbf{b}} \mathbf{v} = \frac{\mathbf{v} \cdot \mathbf{b}}{\|\mathbf{b}\|^2} \mathbf{b}$

Work can be found by $W = \left(\|\mathbf{F}\| \cos\theta\right) \left\|\overrightarrow{PQ}\right\| = \mathbf{F} \cdot \overrightarrow{PQ}$

Cross Products

The cross product is a vector and is defined as $\mathbf{u} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix}$

The cross product is orthogonal to both vectors.

$\|\mathbf{u} \times \mathbf{v}\| = \|\mathbf{u}\| \|\mathbf{v}\| \sin\theta$

The area of a parallelogram is equal to the cross product of the adjacent side vectors.
The cross product is the zero vector if and only if the vectors are parallel.

Triple Scalar Products

The triple scalar product is a scalar and is defined as $\mathbf{u} \cdot \left(\mathbf{v} \times \mathbf{w}\right) = \begin{vmatrix} u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \\ w_1 & w_2 & w_3 \end{vmatrix}$

The absolute value of the triple scalar product is the volume of a parallelpiped.
The triple scalar product is 0 if and only if the vectors are coplanar.

Don't forget to put your name at the top of the page

Chapter 13 - Vector Valued Functions

The derivatives of dot and cross products follow the product rule.

$$(\mathbf{r}_1 \cdot \mathbf{r}_2)' = \mathbf{r}_1 \cdot \mathbf{r}_2' + \mathbf{r}_1' \cdot \mathbf{r}_2 \text{ and } (\mathbf{r}_1 \times \mathbf{r}_2)' = \mathbf{r}_1 \times \mathbf{r}_2' + \mathbf{r}_1' \times \mathbf{r}_2$$

If a vector valued function has constant length, the **r** and **r**' are orthogonal.

The arc length of a smooth vector valued function is $L = \int_a^b \left\| \frac{d\mathbf{r}}{dt} \right\| dt$

Hold down the shift while selecting the ∫ symbol to get it to grow with the integrand.

The chain rule is $\frac{d\mathbf{r}}{d\tau} = \frac{d\mathbf{r}}{dt} \cdot \frac{dt}{d\tau}$

The arc length parametrization is $s = \int_{t_0}^{t} \left\| \frac{d\mathbf{r}}{du} \right\| du$

For a smooth vector valued function, $\left\| \frac{d\mathbf{r}}{dt} \right\| = \frac{ds}{dt}$ and $\left\| \frac{d\mathbf{r}}{ds} \right\| = 1$

To find the unit tangent vector, take the derivative and normalize it. $\mathbf{T}(t) = \frac{\mathbf{r}'(t)}{\|\mathbf{r}'(t)\|}$

To find the unit normal vector, take the derivative of the unit tangent vector and normalize it.

$$\mathbf{N}(t) = \frac{\mathbf{T}'(t)}{\|\mathbf{T}'(t)\|}$$

For curves parametrized by arc length, $\mathbf{T}(s) = \mathbf{r}'(s)$ and $\mathbf{N}(s) = \frac{\mathbf{r}''(s)}{\|\mathbf{r}''(s)\|}$

The binormal vector is the cross product of the unit tangent and unit normal vectors. It is also a unit vector and is found by $\mathbf{B}(t) = \mathbf{T}(t) \times \mathbf{N}(t)$.

The curvature is defined by $\kappa(s) = \left\| \frac{d\mathbf{T}}{ds} \right\| = \|\mathbf{r}''(s)\|$ or $\kappa(t) = \frac{\|\mathbf{T}'(t)\|}{\|\mathbf{r}'(t)\|}$

The radius of the oscillating circle is called the radius of curvature and is $\rho = \frac{1}{\kappa}$

Chapter 14 - Partial Derivatives

The first-order partial derivative of *f* with respect to *x* is denoted by $f_x(x,y)=\dfrac{\partial f}{\partial x}$ and is found by finding the derivative of *f* with every variable other than *x* treated as a constant. The second-order partial derivatives of *f* are $f_{xx}=\dfrac{\partial^2 f}{\partial x^2}$, $f_{yy}=\dfrac{\partial^2 f}{\partial y^2}$ and the mixed partials are $f_{xy}=\dfrac{\partial^2 f}{\partial y \partial x}$, $f_{yx}=\dfrac{\partial^2 f}{\partial x \partial y}$. If *f* is continuous, the $f_{xy}=f_{yx}$. Notice the ordering on the partials. The order for f_{xy} is from left to right, *x* first and *y* second. The order for $\dfrac{\partial^2 f}{\partial x \partial y}$ is right to left, *y* first and *x* second.

If $z=f(x,y)$ is differentiable at (x_0,y_0), then the total differential of *f* at (x_0,y_0) is
$dz=f_x(x_0,y_0)dx+f_y(x_0,y_0)dy$

The chain rule says that if $z=f(x,y)$ and *x* and *y* are both functions of *t*, then
$\dfrac{dz}{dt}=\dfrac{\partial f}{\partial x}\dfrac{dx}{dt}+\dfrac{\partial f}{\partial y}\dfrac{dy}{dt}$. Furthermore, if *x* and *y* are both functions of *u* and *v*, then
$\dfrac{\partial z}{\partial u}=\dfrac{\partial z}{\partial x}\dfrac{\partial x}{\partial u}+\dfrac{\partial z}{\partial y}\dfrac{\partial y}{\partial u}$ and $\dfrac{\partial z}{\partial v}=\dfrac{\partial z}{\partial x}\dfrac{\partial x}{\partial v}+\dfrac{\partial z}{\partial y}\dfrac{\partial y}{\partial v}$

The gradient of *f* is a vector defined by $\nabla f(x,y)=f_x\mathbf{i}+f_y\mathbf{j}$ or
$\nabla f(x,y,z)=f_x\mathbf{i}+f_y\mathbf{j}+f_z\mathbf{k}$. ∇f is read "del f".

The directional derivative of *f* in the direction of the unit vector ***u*** can be written as
$D_{\mathbf{u}}f(x_0,y_0)=\nabla f(x_0,y_0)\cdot\mathbf{u}$

The applications of the gradient are too numerous to fit on this page. It will appear in many formulas.

Chapter 15 - Multiple Integrals

To evaluate a definite integral, work from inside to outside. The order of the integration is important, so be sure to use proper notation.

$$\iint_R f(x,y)\,dA = \int_a^b \int_{g_1(x)}^{g_2(x)} f(x,y)\,dy\,dx = \int_c^d \int_{h_1(y)}^{h_2(y)} f(x,y)\,dx\,dy$$

If you can split the integrand into two independent functions of *x* and *y* and the limits are constant, then you can split a double (or triple) integral up into the product of the integrals.

$$\int_a^b \int_c^d f(x)g(y)\,dy\,dx = \int_a^b f(x)\,dx \int_c^d g(y)\,dy$$

The area of a region R is $A = \iint_R dA$. The volume of a solid G is $V = \iiint_G dV$

The surface area of a parametrically defined surface σ is $S = \iint_R \left\| \frac{\partial \mathbf{r}}{\partial u} \times \frac{\partial \mathbf{r}}{\partial v} \right\| dA$

The center of gravity $(\bar{x}, \bar{y})$ of a lamina is given by $\bar{x} = \frac{M_y}{M} = \frac{1}{\text{mass of R}} \iint_R x\delta(x,y)\,dA$,

$$\bar{y} = \frac{M_x}{M} = \frac{1}{\text{mass of R}} \iint_R y\delta(x,y)\,dA$$

The Theorem of Pappus says that the volume of a solid by revolving a region R about a line L is the area of the region times the distance traveled by the centroid.

For polar and cylindrical coordinates, you need to insert an extra r into the integrand. For spherical coordinates, you need to insert an extra $\rho^2 \sin\phi$ into the integrand. All of this is related to the Jacobian. If T is a transformation from the uv plane into the xy plane, then the Jacobian of T is denoted by $J(u,v) = \frac{\partial(x,y)}{\partial(u,v)} = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial y}{\partial u} \\ \frac{\partial x}{\partial v} & \frac{\partial y}{\partial v} \end{vmatrix}$. When applying the transformation, multiply the integrand by the absolute value of the Jacobian.

Chapter 16 - Topics in Vector Calculus

Consider the vector function $\mathbf{F}(x,y,z) = f(x,y,z)\mathbf{i} + g(x,y,z)\mathbf{j} + h(x,y,z)\mathbf{k}$.

The divergence of F is a scalar defined by $\operatorname{div}\mathbf{F} = \dfrac{\partial f}{\partial x} + \dfrac{\partial g}{\partial y} + \dfrac{\partial h}{\partial z} = \nabla \cdot \mathbf{F}$

The curl of F is a vector defined by $\operatorname{curl}\mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ f & g & h \end{vmatrix} = \nabla \times \mathbf{F}$

The line integral of *f* with respect to *s* along *C* is the net signed area between the curve *C* and the graph of $f(x,y)$ and is denoted by $A = \int_C f(x,y)\,ds$.

Arc length can be expressed as $L = \int_C ds$.

The value of a line integral does not depend on its parametrization. However, if the orientation is reversed, the sign of the integral with respect to *x* and *y* changes, but the integral with respect to the arc length parameter *s* remains unchanged.

The work performed by the vector field is $W = \int_C \mathbf{F}\cdot\mathbf{T}\,ds = \int_C \mathbf{F}\cdot d\mathbf{r}$

If F is a conservative vector field and $\mathbf{F}(x,y) = \nabla\phi(x,y)$ then the first fundamental theorem of calculus applies to line integrals and $\int_C \mathbf{F}(x,y)\cdot d\mathbf{r} = \phi(x_1,y_1) - \phi(x_2,y_2)$

If F is a conservative vector field, then $\int_C \mathbf{F}\cdot d\mathbf{r} = 0$ for every piecewise smooth closed curve C and the integral is independent of the path.

A vector field in 2-space is conservative if $\dfrac{\partial f}{\partial y} = \dfrac{\partial g}{\partial x}$ and in 3-space if $\operatorname{curl}\mathbf{F} = \mathbf{0}$

Green's Theorem says $\oint_C f(x,y)\,dx + g(x,y)\,dy = \iint\limits_R \left(\dfrac{\partial g}{\partial x} - \dfrac{\partial f}{\partial y}\right) dA$

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