Relative Maximums and Minimums - Problem 14.8.21

The original function f(x,y)

#1: $2\cdot x^2 - 4\cdot x\cdot y + y^4 + 2$

Now put a z in front of it so that we can create level curves. **z=#1**. Just hit enter, not control-enter since there is no simplification to do.

#2: $z = 2\cdot x^2 - 4\cdot x\cdot y + y^4 + 2$

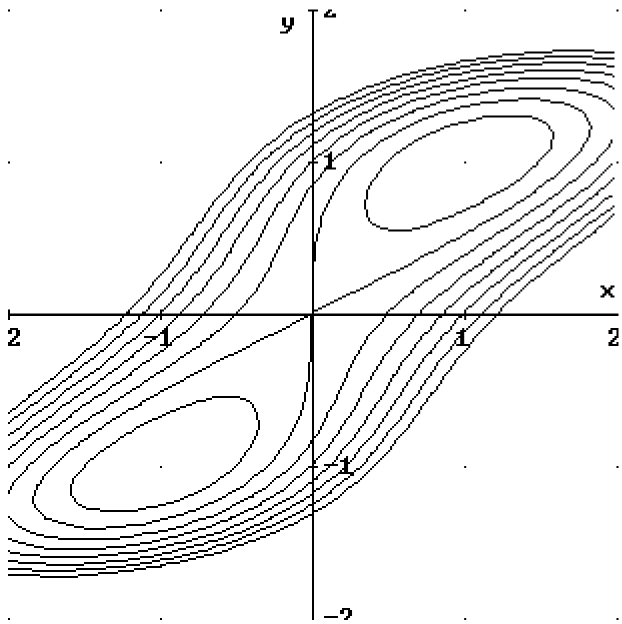
Generate a contour plot by making a vector and simplying it. I'm going to try -5 to 5 in increments of 0.5. Some of these may not graph and I may need to change the levels to get a better picture of what's going on.

#3: $\text{VECTOR}(z = 2\cdot x^2 - 4\cdot x\cdot y + y^4 + 2, z, -5, 5, 0.5)$

#4: $\Big[-5 = 2\cdot x^2 - 4\cdot x\cdot y + y^4 + 2, -\frac{9}{2} = 2\cdot x^2 - 4\cdot x\cdot y + y^4 + 2, -4 = 2\cdot x^2 - 4\cdot x\cdot y + y^4 + 2, -\frac{7}{2} = 2\cdot x^2 - 4\cdot x\cdot y + y^4 + 2, -3 = 2\cdot x^2 - 4\cdot x\cdot y + y^4 + 2, -\frac{5}{2} = 2\cdot x^2 - 4\cdot x\cdot y + y^4 + 2, -2 = 2\cdot x^2 - 4\cdot x\cdot y + y^4 + 2, -\frac{3}{2} = 2\cdot x^2 - 4\cdot x\cdot y + y^4 + 2, -1 = 2\cdot x^2 - 4\cdot x\cdot y + y^4 + 2, -\frac{1}{2} = 2\cdot x^2 - 4\cdot x\cdot y + y^4 + 2, 0 = 2\cdot x^2 - 4\cdot x\cdot y + y^4 + 2, \frac{1}{2} = 2\cdot x^2 - 4\cdot x\cdot y + y^4 + 2, 1 = 2\cdot x^2 - 4\cdot x\cdot y + y^4 + 2, \frac{3}{2} = 2\cdot x^2 - 4\cdot x\cdot y + y^4 + 2, 2 = 2\cdot x^2 - 4\cdot x\cdot y + y^4 + 2, \frac{5}{2} = 2\cdot x^2 - 4\cdot x\cdot y + y^4 + 2, 3 = 2\cdot x^2 - 4\cdot x\cdot y + y^4 + 2, \frac{7}{2} = 2\cdot x^2 - 4\cdot x\cdot y + y^4 + 2, 4 =$

$$2\cdot x^2 - 4\cdot x\cdot y + y^4 + 2,\ \frac{9}{2} = 2\cdot x^2 - 4\cdot x\cdot y + y^4 + 2,\ 5 = 2\cdot x^2 - 4\cdot x\cdot y + y^4 + 2\Big]$$

Now Insert a 2D Plot and then insert a plot of expression #4



It looks like something is going on at (1,1) and (-1,-1). There might be something at (0,0), we'll wait and see what the calculus says. From this contour plot, I don't know whether the points are maximums or minimums since there is no shading and I don't know which level corresponds to which plot. If I change the vector and only plot a certain rain, I might be able to figure it out, but we can also use calculus to find the values.

Find the first order partial derivatives.

#5: $\frac{d}{dx}(2\cdot x^2 - 4\cdot x\cdot y + y^4 + 2)$

#6: $4\cdot x - 4\cdot y$

#7: $\frac{d}{dy}(2\cdot x^2 - 4\cdot x\cdot y + y^4 + 2)$

#8: $4\cdot y^3 - 4\cdot x$

Find the critical points by solving the system of linear equations. Create the system by grouping #6 and #8 together **[#6,#8]** and then using the solve command. Solve for both x and y.

#9: $\left[4\cdot x - 4\cdot y,\ 4\cdot y^3 - 4\cdot x\right]$

#10: $\mathrm{SOLVE}(\left[4\cdot x - 4\cdot y,\ 4\cdot y^3 - 4\cdot x\right], [x, y])$

#11: $[x = 0 \wedge y = 0,\ x = 1 \wedge y = 1,\ x = -1 \wedge y = -1]$

We find the critical points are (0,0), (1,1) and (-1,-1).

Go ahead and find the z-values for each of the critical points by substituting x and y into expression #1. To do this, enter the x value, then click on the y and enter the y value; after both values are entered, click simplify.

#12: 2

#13: 1

#14: 1

Now let's move on and find the second partials. Since the first partials are in #6 and #8, we'll find the first order partial derivatives of those to find the second order partials.

#15: $\frac{d}{dx}(4\cdot x - 4\cdot y)$

#16: 4

#17: $\frac{d}{dy}(4\cdot x - 4\cdot y)$

#18: -4

#19: $\frac{d}{dx}(4\cdot y^3 - 4\cdot x)$

#20: -4

#21: $\frac{d}{dy}(4\cdot y^3 - 4\cdot x)$

#22: $12\cdot y^2$

Notice that the mixed second order partials are the same which is what we would expect since they are continuous.

Now we find D = fxx*fyy - fxy*fyx. **#16*#22-#18*#20**

#23: $4\cdot(12\cdot y^2) - (-4)\cdot(-4)$

#24: $48\cdot y^2 - 16$

Evaluate D for each critical point (0,0), (1,1), and (-1,-1)

#25: -16

At (0,0), D = -16 < 0, so there is a saddle point at (0,0,2)

#26: 32

At (1,1), D = 32 > 0, so we look at fxx = 4 > 0. This means that there is a relative minimum at (1,1,1).

#27: 32

At (-1,-1), D = 32 > 0, so we look at fxx = 4 > 0. This means that there is a relative minimum at (-1,-1,1). Notice that since fxx is always 4, which is positive, it is impossible for this function to have a relative maximum.

Now let's annotate the graph and add the points to it. Copy and paste the graph from above and then double click it to edit it and make changes. Use the Insert / Annotation (F12) key to add comments to the graph. (Hint: Create another expression that can be deleted - example #28: 1. Then paste the graph in after that. I had problems pasting when a comment was active and a comment was the last thing in a file. After pasting, delete expression #28.)