Partial Derivatives

Technology Exercise 3
20 points

Use Derive to find the following. Annotate your output using Text Objects (F5).

1. Graph the level curve $z = k$ for the specified values of *k*. To do this, enter the equation and then go to Calculus / Vector and create a vector where *z* ranges over the values of *k*. Insert a 2D plot and then plot the vector.

   a. $z = x^2 + 9y^2;\ k = 0, 1, 2, 3, 4$

   b. $z = y \csc x;\ k = -2, -1, 0, 1, 2$

2. Use the LIM2 function (see help) to find $\lim_{(x,y)\to(3,2)} \dfrac{2x - 3y}{8x^3 - 27y^3}$

3. Consider the function $f(x, y) = 2xy^2 - x^2y + 4xy$.

   a. Generate a contour plot with a viewing window of [-3,8]×[-5,3] and use the plot to approximate the locations of all relative extrema and saddle points in the region.

   b. Find the first partial derivatives of *f* and find the critical points.

   c. Find the second partial derivatives and evaluate them at each critical point to determine whether the critical point is a relative maximum, relative minimum, saddle point, or no conclusion can be drawn. Go back to your graph from part a. and label the points.

4. Use the GRAD function to find a unit vector in the direction in which $f(x, y, z) = \dfrac{x}{z} + \dfrac{z}{y^2}$ increases and decreases most rapidly at the point P(1,2,-2). Find the rate of change in each of those directions.