Vector-Valued Functions

Technology Exercise 2

20 points

Use Derive to find the following. Annotate your output using Text Objects (F5).

1. Consider the vector-valued function $\mathbf{r}(t) = t\,\mathbf{i} + \frac{1}{2}t^2\,\mathbf{j} + \frac{1}{3}t^3\,\mathbf{k}$

   a. Use Derive to find $\mathbf{T}(t)$, $\mathbf{N}(t)$, and $\mathbf{B}(t)$ when $t = 1$. Annotate the output with labels so it is clear what the output is representing.

   b. Insert a 3D graph of the vector valued function and draw the unit tangent, normal, and binormal vectors at the point when $t = 1$. Find a suitable view and annotate the graph with **r**, **T**, **N**, and **B**. Update the graph. Copy the graph and find another good view. Unfortunately, the labels don't rotate with the view, so you will have to drag your vector labels to the proper locations.

   c. Find the curvature and radius of curvature at $t = 1$.

2. Use Derive to find the curvature and the radius of curvature at the stated point.

   a. $\mathbf{r}(t) = \sin t\,\mathbf{i} + \cos t\,\mathbf{j} + \frac{1}{2}t^2\,\mathbf{k};\ t = \frac{\pi}{3}$

   b. $y = e^{-x},\ x = 1$

3. Consider the curve $\mathbf{r}(t) = \sin e^t\,\mathbf{i} + \cos e^t\,\mathbf{j} + e^t\sqrt{3}\,\mathbf{k};\ t \geq 0$

   a. Find an arc length parametrization of the curve $\mathbf{r}(t)$ that has the same orientation as the given curve and has $t = 0$ as the reference point.

   b. Find $\mathbf{T}(s)$, $\mathbf{N}(s)$, and $\mathbf{B}(s)$.