# Vector Calculus

Technology Exercise 5
20 points

Use Derive to find the following. Annotate all output.

1. Find the divergence and curl for $\mathbf{F}(x,y,z) = e^{xz}\,\mathbf{i} + 3xe^{y}\,\mathbf{j} - e^{yz}\,\mathbf{k}$

2. Evaluate $\displaystyle\int_C \frac{e^{-z}}{x^2+y^2}\,ds$ where
$C: x = 2\cos t,\ y = 2\sin t,\ z = t\ \ (0 \le t \le \pi)$

3. Show that the vector-valued function **F** is conservative and then find its potential function $\phi(x,y,z)$.
$$\mathbf{F}(x,y,z) = \left(3y^2z - 5y^2 - 14xz\right)\mathbf{i} + \left(6xyz - 10xy + 2z^3\right)\mathbf{j} + \left(3xy^2 + 6yz^2 - 7x^2\right)\mathbf{k}$$

4. Use Green's Theorem to evaluate $\oint_C x^2y\,dx + \left(y + xy^2\right)dy$ where C is the boundary of the region enclosed by $y = x^2$ and $x = y^2$. Assume that C is oriented clockwise.

5. Set up an iterated integral equal to $\iint_\sigma xyz\,dS$ by projecting σ onto (a) the xy-plane, (b) the yz-plane, and (c) the xz-plane. Then evaluate each integral and show they are equivalent. σ is the portion of the plane $2x + 3y + 4z = 12$ in the first octant.

6. Find the flux of the vector field $\mathbf{F}(x,y,z) = e^{-y}\,\mathbf{i} - y\,\mathbf{j} + x\sin z\,\mathbf{k}$ across the surface σ in the direction of positive orientation. σ is the portion of the paraboloid $\mathbf{r}(u,v) = u\cos v\,\mathbf{i} + u\sin v\,\mathbf{j} + \left(1-u^2\right)\mathbf{k}$ with $1 \le u \le 2,\ 0 \le v \le 2\pi$

7. Use the Divergence theorem to evaluate $\iint\limits_{\sigma} \mathbf{F}\cdot\mathbf{n}\,dS$ where $\mathbf{F}(x,y,z) = xy\,\mathbf{i} + yz\,\mathbf{j} + xz\,\mathbf{k}$ and σ is the surface of the cube bounded by the planes $x=0,\ x=2,\ y=0,\ y=2,\ z=0,\ z=2$.

8. Use Stoke's Theorem to evaluate the integral $\oint_C \mathbf{F}\cdot d\mathbf{r}$, where $\mathbf{F}(x,y,z) = -3y^2\,\mathbf{i} + 4z\,\mathbf{j} + 6x\,\mathbf{k}$ and C is the triangle in the $z = \frac{1}{2}y$ plane with vertices at (2,0,0), (0,2,1), and (0,0,0) with a counterclockwise orientation looking down the positive z axis.