Mathematical Notation

Math 116 - College Algebra

Purpose:
One goal in any course is to properly use the language of that subject. Algebra is no different and may often seem like a foreign language. These notations summarize some of the major concepts and more difficult topics of the unit. Typing them helps you learn the material while teaching you to properly express mathematics on the computer. Part of your grade is for *properly* using mathematical content.

Instructions:
Use Word or WordPerfect to recreate the following documents. Each article is worth 10 points and should be emailed to the instructor at james@richland.edu. This is not a group assignment, each person needs to create and submit their own notation.

Type your name at the top of each document. Include the title as part of what you type. The lines around the title aren't that important, but if you will type ----- at the beginning of a line and hit enter, both Word and WordPerfect will convert it to a line.

For expressions or equations, you should use the equation editor in Word or WordPerfect. The instructor used WordPerfect and a 14 pt Times New Roman font with 0.75" margins, so they may not look exactly the same as your document. The equations were created using 14 pt font and appear a little larger than normal text. This is a quirk of WordPerfect, but it allows you to see what was done with the equation editor versus the word processor.

If there is an equation, put both sides of the equation into the same equation editor box instead of creating two objects. Be sure to use the proper symbols, there are some instances where more than one symbol may look the same, but they have different meanings and don't appear the same as what's on the assignment.

There are instructions on how to use the equation editor in a separate document or on the website. Be sure to read through the help it provides. There are some examples at the end that walk students through the more difficult problems. You will want to read the handout on using the equation editor if you have not used this software before.

If you fail to type your name on the document, you will lose 1 point. Don't type the hints or reminders that appear on the pages.

These notations are due before the beginning of class on the day of the exam for that material. For example, notation 1 is due on the day of exam 1. Late work will be accepted but will lose 20% of its value per class period. If I receive your emailed assignment more than one class period before it is due and you don't receive all 10 points, then I will email you back with things to correct so that you can get all the points. Any corrections need to be submitted by the due date and time or the original score will be used.

Remember to put your name at the top of the page.

Chapter 1 - Shifting, Reflecting, and Stretching Graphs

Consider the function defined by $g(x) = a \cdot f\left(\frac{x-c}{b}\right) + d$.

The shifting transformation is a translation where the graph retains its size and shape but its position is changed. Shifts occur because of addition or subtraction, so the two variables in the above function that would cause shifts are *c* and *d*.

A scaling transformation is one where the size of the graph is changed. Because the size is changed, the position is often changed with it. Scaling occurs because of multiplication or division, so the two variables in the above function that would cause scaling are *a* and *b*.

A reflection is a transformation is a special case of scaling where the constant is -1. The graph retains its size and shape but is reflected about an axis.

The constants grouped with the *x* (*b* and *c*) are horizontal transformations, that is, they affect the *x* but not the *y*. Also notice that they act backwards from what you might think (hence the subtraction and division). The constants that are not grouped with the *x* (*a* and *d*) affect the *y* but not the *x*. They are vertical transformations and act the way you would expect.

The following chart illustrates some examples.

Graph	**Translation**	**Domain**	**Range**
$y = f(x)$	None	[3, 6)	(1, 5]
$y = 3f(x-2)$	Multiply y's by 3, Right 2	[5, 8)	(3, 15]
$y = 2 - f(3x+6)$	Reflect about the x-axis, up 2, divide x's by 3, left 2	[-1, 0)	[-3, 1)
$y = f(-x) + 4$	Reflect about y-axis, up 4	(-6, -3]	(5, 9]
$y = 1/f(5-x) + 2$	Take reciprocals of y's, up 2; reflect about y-axis, right 5	(-1, 2]	[11/5, 3)

Remember to put your name at the top of the page.

Chapter 2 - Solving Equations Algebraically

Complete the square to solve $x^2 - 4x + 7 = 0$.

$$\begin{aligned} x^2 - 4x + 7 &= 0 \\ x^2 - 4x &= -7 \\ x^2 - 4x + 4 &= -7 + 4 \\ (x-2)^2 &= -3 \\ x - 2 &= \pm\sqrt{-3} \\ x &= 2 \pm i\sqrt{3} \end{aligned}$$

Note: the line spacing was changed from 150% to 100% for this chapter and the equations are aligned at the = sign. Don't type this note on your paper.

When solving an equation involving radicals, isolate the radical and then square both sides. Be sure to check for extraneous solutions.

$$\begin{aligned} \sqrt{x+1} - 3x &= 1 \\ \sqrt{x+1} &= 3x + 1 \\ \left(\sqrt{x+1}\right)^2 &= (3x+1)^2 \\ x + 1 &= 9x^2 + 6x + 1 \\ 9x^2 + 5x &= 0 \\ x(9x+5) &= 0 \end{aligned}$$

There are two answers, $x = 0$ and $x = -5/9$, but only $x = 0$ checks. The answer is $x = 0$.

When factoring, always factor out the greatest common factor first. The greatest common factor is the factor with the smallest exponents.

$$\begin{aligned} 3x(x-1)^{1/2} + 2(x-1)^{3/2} &= 0 \\ (x-1)^{1/2}\left[3x + 2(x-1)\right] &= 0 \\ (x-1)^{1/2}(5x-2) &= 0 \end{aligned}$$

There are two answers, $x = 1$ and $x = 2/5$, but only $x = 1$ is in the domain. The one-half power means the square root, so there is really a $\sqrt{x-1}$ in the problem and the implied domain is $x \geq 1$. Since $x = 2/5$ doesn't fall in that domain, it can't be used as an answer. The answer is $x = 1$.

Chapter 3 - Polynomials

Consider the polynomial $f(x) = -2(x+3)^2(x-5)^3(x+4)$.

If it were multiplied out, the leading term would be $-2 \cdot x^2 \cdot x^3 \cdot x = -2x^6$ and the constant would be $f(0) = -2(3)^2(-5)^3(4) = 9000$.

Because the degree of the polynomial is 6, there will be exactly 6 real or complex roots and a maximum of 5 turns.

On the far right side of the graph, as $x \to +\infty, \ y \to -\infty$ because the leading coefficient is negative. On the far left side of the graph, as $x \to -\infty, \ y \to -\infty$ because the degree is even so the left side will do the same as the right side.

The rational root theorem says that any rational roots will be of the form of a factor of 9000 over a factor of 2.

The graph of the function will cross the *x*-axis as $x = 5$ and $x = -4$ because the corresponding factors have odd exponents. The graph will touch, but not cross, the *x*-axis at $x = -3$ because the corresponding factor has an even exponent.

When the coefficients are real, then complex solutions come in pairs. If $a + bi$ is a root, then so is $a - bi$.

When the coefficients are rational, then irrational solutions involving square roots come in pairs. If $a + \sqrt{b}$ is a root, then so is $a - \sqrt{b}$.

Chapter 4 - Exponential and Logarithmic Functions

A logarithm is an exponent.

Conversion between exponential form and logarithmic form

$$x = a^y \Leftrightarrow y = \log_a x$$

Properties of Logarithms

$$\log_a 1 = 0$$
$$\log_a a = 1$$
$$\log_a a^x = x$$
$$a^{\log_a x} = x, \;\; x > 0$$

The log of a product is the sum of the logs

$$\log_a xy = \log_a x + \log_a y$$

The log of a quotient is the difference of the logs

$$\log_a \frac{x}{y} = \log_a x - \log_a y$$

The exponent on the argument of the log is the coefficient of the log

$$\log_a x^n = n \cdot \log_a x$$

Change of base formula

$$\log_a x = \frac{\log_b x}{\log_b a} = \frac{\log x}{\log a} = \frac{\ln x}{\ln b}$$

Chapter 7 - Conics

Determining the Type of Conic

Circle	$x^2 + y^2 = 9$
Ellipse	$9x^2 + 4y^2 = 144$
Point	$4x^2 + 9y^2 = 0$
No Graph	$3x^2 + 7y^2 = -4$
Hyperbola	$5x^2 - 4y^2 = 18$
Intersecting Lines	$4x^2 - 3y^2 = 0$
Parabola	$x^2 - 5y + 7 = 0$
Parallel Lines	$x^2 - x - 6 = 0$
Line	$3x + 4y = 12$

Standard Forms

Conic	Horizontal	Vertical
Parabola	$y^2 = 4px$	$x^2 = 4py$
Ellipse	$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$	$\frac{y^2}{a^2} + \frac{x^2}{b^2} = 1$
Hyperbola	$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$	$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$

Chapter 5 - Systems of Linear Equations

A system of linear equations can be written as an augmented matrix where each equation is a row and each variable is a column.

Note: the system was written using a right aligned 2×5 matrix with the column spacing set to 50% . Don't type this note on your paper.

$$\begin{cases} x - 2y = 7 \\ 3x + y = 9 \end{cases} \text{ becomes } \left[\begin{array}{rr|r} 1 & -2 & 7 \\ 3 & 1 & 9 \end{array}\right]$$

Elementary Row Operations

There are three operations you can perform to a matrix that will not change the solution. You can interchange two rows, multiply a row by a non-zero constant, or add a constant multiple of one row to another row.

Row Echelon Form – Gaussian Elimination

A matrix is in row echelon form when any row of zeros is at the bottom, the first non-zero element of any row is a one, and the leading one of any row is to the right of the leading one of the previous row. A matrix can be placed into row echelon form by using Gaussian Elimination and the solution read using back substitution.

$$\left[\begin{array}{rr|r} 1 & 7 & 2 \\ 0 & 1 & 3 \end{array}\right]$$

Reduced Row Echelon Form – Gauss-Jordan Elimination

A matrix is in reduced row echelon form when it is in row echelon form and any element above or below a leading one is a zero. A matrix can be placed into reduced row echelon form by using Gauss-Jordan Elimination and there is no back substitution needed to read the solution.

$$\left[\begin{array}{rrr|r} 1 & 0 & 5 & 1 \\ 0 & 1 & 2 & 6 \end{array}\right]$$

Solving a System using Matrix Inverses

A system of equations can be written as $\mathbf{AX} = \mathbf{B}$ where **A** is the coefficient matrix, **X** is the variable matrix, and **B** is the constant matrix (right hand side). If **A** is a square matrix and has an inverse, the solution is $\mathbf{X} = \mathbf{A}^{-1}\mathbf{B}$.

Remember to put your name at the top of the page.

Chapter 6 - Sequences and Series

Arithmetic Sequences & Series (Linear)

Common Difference: $d = a_{n+1} - a_n$

General Term: $a_n = a_1 + (n-1)d$

n^{th} partial sum: $S_n = \frac{n}{2}(a_1 + a_n)$

Geometric Sequences & Series (Exponential)

Common Ratio: $r = \frac{a_{n+1}}{a_n}$

General Term: $a_n = a_1 \cdot r^{n-1}$

n^{th} partial sum: $S_n = \frac{a_1(1-r^n)}{1-r}$

Infinite sum: $S = \frac{a_1}{1-r},\ |r| < 1$

Binomial Expansion Theorem

$$(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^{n-k} y^k$$

$$\begin{aligned}(x-2)^4 &= \binom{4}{0} x^4 (-2)^0 + \binom{4}{1} x^3 (-2)^1 + \binom{4}{2} x^2 (-2)^2 \\ &\quad + \binom{4}{3} x^1 (-2)^3 + \binom{4}{4} x^0 (-2)^4 \\ &= 1x^4(1) + 4x^3(-2) + 6x^2(4) + 4x(-8) + 1(1)(16) \\ &= x^4 - 8x^3 + 24x^2 - 32x + 16\end{aligned}$$

The example $(x-2)^4$ is all one object and was aligned at the =. I split the first line because it was too long for one line with my font, but you may get it to fit on one line if you're using a smaller font.

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