Mathematical Notation

Math 122 - Calculus & Analytic Geometry II

Purpose:
One goal in any course is to properly use the language of that subject. Calculus is no different and may often seem like a foreign language. These notations summarize some of the major concepts and more difficult topics of the unit. Typing them helps you learn the material while teaching you to properly express mathematics on the computer. Part of your grade is for *properly* using mathematical content.

Instructions:
Use Word or WordPerfect to recreate the following documents. Each article is worth 10 points and should be emailed to the instructor at james@richland.edu. This is not a group assignment, each person needs to create and submit their own notation.

Type your name at the top of each document. Include the title as part of what you type. The lines around the title aren't that important, but if you will type ----- at the beginning of a line and hit enter, both Word and WordPerfect will convert it to a line.

For expressions or equations, you should use the equation editor in Word or WordPerfect. The instructor used WordPerfect and a 14 pt Times New Roman font with 0.75" margins, so they may not look exactly the same as your document. The equations were created using 14 pt font and appear a little larger than normal text. This is a quirk of WordPerfect, but it allows you to see what was done with the equation editor versus the word processor.

If there is an equation, put both sides of the equation into the same equation editor box instead of creating two objects. Be sure to use the proper symbols, there are some instances where more than one symbol may look the same, but they have different meanings and don't appear the same as what's on the assignment.

There are instructions on how to use the equation editor in a separate document or on the website. Be sure to read through the help it provides. There are some examples at the end that walk students through the more difficult problems. You will want to read the handout on using the equation editor if you have not used this software before.

If you fail to type your name on the document, you will lose 1 point. Don't type the hints or reminders that appear on the pages.

These notations are due before the beginning of class on the day of the exam for that material. For example, notation 1 is due on the day of exam 1. Late work will be accepted but will lose 20% of its value per class period. If I receive your emailed assignment more than one class period before it is due and you don't receive all 10 points, then I will email you back with things to correct so that you can get all the points. Any corrections need to be submitted by the due date and time or the original score will be used.

Remember to put your name at the top of the page.

Chapter 7 - Inverse Functions

$$\frac{d}{dx}\left[f^{-1}(x)\right]=\frac{1}{f'\left[f^{-1}(x)\right]} \qquad \frac{dy}{dx}=\frac{1}{dx/dy}$$

$$e=\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^{x}=\lim_{x\to 0}\left(1+x\right)^{1/x}$$

$$\frac{d}{dx}\left[e^{x}\right]=e^{x} \qquad \frac{d}{dx}\left[a^{x}\right]=a^{x}\cdot\ln a \qquad \frac{d}{dx}\left[\ln x\right]=\frac{1}{x} \qquad \frac{d}{dx}\left[\log_a x\right]=\frac{1}{x\cdot\ln a}$$

$$\frac{d}{dx}\left[\sin^{-1}x\right]=\frac{1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}\left[\cos^{-1}x\right]=\frac{-1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}\left[\tan^{-1}x\right]=\frac{1}{1+x^2} \qquad \frac{d}{dx}\left[\cot^{-1}x\right]=\frac{-1}{1+x^2}$$

$$\frac{d}{dx}\left[\sec^{-1}x\right]=\frac{1}{|x|\sqrt{x^2-1}} \qquad \frac{d}{dx}\left[\csc^{-1}x\right]=\frac{-1}{|x|\sqrt{x^2-1}}$$

If f and g are differentiable and $\lim_{x\to a} f(x)$ and $\lim_{x\to a} g(x)$ are both 0 or ∞, then

$$\lim_{x\to a}\left(\frac{f(x)}{g(x)}\right)=\lim_{x\to a}\left(\frac{f'(x)}{g'(x)}\right)$$

$$\frac{d}{dx}\left[\sinh x\right]=\cosh x \qquad \frac{d}{dx}\left[\cosh x\right]=\sinh x$$

$$\frac{d}{dx}\left[\tanh x\right]=\operatorname{sech}^2 x \qquad \frac{d}{dx}\left[\coth x\right]=-\operatorname{csch}^2 x$$

$$\frac{d}{dx}\left[\operatorname{sech} x\right]=-\operatorname{sech} x\tanh x \qquad \frac{d}{dx}\left[\operatorname{csch} x\right]=-\operatorname{csch} x\coth x$$

Chapter 8 - Techniques of Integration

Integration by Parts

Split the original integrand into two parts, *u* and *dv*. *dv* must be integrable and *du* should be simpler than *u*. Good choices for *u* follow the LIATE rule.

Since $d[uv] = u\,dv - v\,du$, we can write $\int u\,dv = uv - \int v\,du$.

Reduction Formulas

$$\int \sin^n x\,dx = -\frac{1}{n}\sin^{n-1} x\cos x + \frac{n-1}{n}\int \sin^{n-2} x\,dx$$

$$\int \cos^n x\,dx = \frac{1}{n}\cos^{n-1} x\sin x + \frac{n-1}{n}\int \cos^{n-2} x\,dx$$

$$\int \tan^n x\,dx = \frac{1}{n-1}\tan^{n-1} x - \int \tan^{n-2} x\,dx$$

$$\int \sec^n x\,dx = \frac{1}{n-1}\sec^{n-2} x\tan x + \frac{n-2}{n-1}\int \sec^{n-2} x\,dx$$

Trigonometric Substitution

For $\sqrt{a^2 - x^2}$, use $x = a\sin\theta$

For $\sqrt{a^2 + x^2}$, use $x = a\tan\theta$

For $\sqrt{x^2 - a^2}$, use $x = a\sec\theta$

Special Trigonometric Substitution

$$\sin x = \frac{2u}{1+u^2} \qquad \cos x = \frac{1-u^2}{1+u^2} \qquad dx = \frac{2}{1+u^2}du$$

Errors with Numerical Integration

$$|E_M| \le \frac{(b-a)^3 K_2}{24n^2} \qquad |E_T| \le \frac{(b-a)^3 K_2}{12n^2} \qquad |E_S| \le \frac{(b-a)^5 K_4}{180n^4}$$

Remember to put your name at the top of the page.

Chapter 9 - Differential Equations

First order, linear differential equations can be written in the form
$\frac{dy}{dx} + p(x)y = q(x)$ or $y' + p(x)y = q(x)$.

The integrating factor for a first-order, linear differential equation is $\mu = e^{\int p(x)dx}$. After finding the integrating factor, multiply both sides of the equation by it to get $\mu y' + \mu p(x)y = \mu q(x)$. The left side of the equation is the derivative of a product and so the equation can be written as $\frac{d}{dx}[\mu y] = \mu q(x)$. This equation is separable and can then be integrated to find the solution.

Models where the rate of growth (decay) is proportional to the amount present can be written as $\frac{dy}{dt} = ky$ and give the model $y = Ce^{kt}$.

Models where the rate of growth is proportional to the amount present and the amount not present can be written as $\frac{dy}{dt} = ky(L-y)$ and give the model $y = \frac{L}{1+Ce^{-kt}}$.

The solution to a second order, homogenous linear differential equation $y'' + py' + q = 0$ can be found by finding the roots m_1 and m_2 to the equation $m^2 + pm + q = 0$ and then using the form $y(x) = c_1e^{m_1x} + c_2e^{m_2x}$, $y(x) = c_1e^{mx} + c_2xe^{mx}$, or $y(x) = e^{ax}(c_1\cos bx + c_2\sin bx)$.

Chapter 10 - Infinite Series

Maclaurin polynomial

$$p_n(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 + \cdots + \frac{f^{(n)}(0)}{n!}x^n$$

$$p_n(x) = \sum_{k=0}^{n} \frac{f^{(k)}(0)}{k!}x^k$$

Taylor Polynomial about $x = a$

$$p_n(x) = \sum_{k=0}^{n} \frac{f^{(k)}(a)}{k!}(x-a)^k$$

$\{a_n\}$ is increasing if $a_{n+1} - a_n \geq 0,\ \dfrac{a_{n+1}}{a_n} \geq 1,$ or $f'(x) \geq 0$ and decreasing if $a_{n+1} - a_n \leq 0,\ \dfrac{a_{n+1}}{a_n} \leq 1,$ or $f'(x) \leq 0$.

The sum of an infinite geometric series is $\displaystyle\sum_{k=0}^{\infty} ar^k = \frac{a}{1-r},\ |r| < 1$

Every power series will either converge at a point, converge for all real numbers, or converge on a finite open interval.

Common Maclaurin series

$$e^x = \sum_{k=0}^{\infty} \frac{x^k}{k!} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$$

$$\cos x = \sum_{k=0}^{\infty} (-1)^k \frac{x^{2k}}{(2k)!} = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots$$

$$(1+x)^m = 1 + \sum_{k=1}^{\infty} \frac{m(m-1)\cdots(m-k+1)}{k!}x^k$$

Chapter 11 - Analytic Geometry

$$x = r\cos\theta \quad y = r\sin\theta \quad r^2 = x^2 + y^2 \quad \tan\theta = \frac{y}{x}$$

$$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{r\cos\theta + \sin\theta\dfrac{dr}{d\theta}}{-r\sin\theta + \cos\theta\dfrac{dr}{d\theta}}$$

$$L = \int_{\alpha}^{\beta} \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta \qquad A = \int_{\alpha}^{\beta} \frac{1}{2} r^2\, d\theta$$

Standard Forms for Conic Sections

Conic	Horizontal	Vertical
Parabola	$y^2 = 4px$	$x^2 = 4py$
Ellipse	$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$	$\frac{y^2}{a^2} + \frac{x^2}{b^2} = 1$
Hyperbola	$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$	$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$

Eliminating the xy-term

Choose an angle θ such that $\cot 2\theta = \dfrac{A - C}{B}, \ 0^\circ < 2\theta < 180^\circ$

Polar Equation of Conics

Relation of Directrix to Pole

Right	Left	Above	Below
$r = \frac{ed}{1 + e\cos\theta}$	$r = \frac{ed}{1 - e\cos\theta}$	$r = \frac{ed}{1 + e\sin\theta}$	$r = \frac{ed}{1 - e\sin\theta}$

e = eccentricity (Ellipse if $0 < e < 1$, Parabola if $e = 1$, Hyperbola if $e > 1$)

d = distance from pole to directrix.

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