Example Technology Excercise 3 John Smith and Tom Brown

1 Illustrate Mean Value Theorem

Declare the function and the endpoints

```
(%i1) Y(x):=1/4*x*(x-2)*(x+1);
      a:-2;
      b:3;
```

$$(\%o1)\quad Y(x):=\frac{1}{4}x\left(x-2\right)\left(x+1\right)$$

$$(\%o2)\quad -2$$

$$(\%o3)\quad 3$$

Find the slope of the secant line

```
(%i4) msec:(Y(b)-Y(a))/(b-a);
```

$$(\%o4)\quad 1$$

Find the derivative and see where it equals the slope of the secant line

```
(%i5) diff(Y(x),x),factor;
      sol:solve(%=msec,x);
```

$$(\%o5)\quad \frac{3x^2-2x-2}{4}$$

$$(\%o6)\quad \left[x=-\frac{\sqrt{19}-1}{3},x=\frac{\sqrt{19}+1}{3}\right]$$

Find y-intercepts of tangent lines so we can graph them
Since y = mx+b, b = y-mx

```
(%i7) x1:rhs(sol[1])$
      b1:Y(x1)-msec*x1,float;
      x2:rhs(sol[2])$
      b2:Y(x2)-msec*x2,float;
```

(%o8) 1.015168146801348

(%o10) -2.052205183838385

The tangent lines are

```
(%i11) y=msec*x+b1;
       y=msec*x+b2;
```

(%o11) $y = x + 1.015168146801348$

(%o12) $y = x - 2.052205183838385$

2 Solve these problems

2.1 Problem 3.4.43

Define the function Y and its first two derivatives

```
(%i13) Y:x^4-4*x^3+2;
       Y1:diff(Y,x);
       Y2:diff(Y,x,2);
```

(%o13) $x^4 - 4\,x^3 + 2$

(%o14) $4\,x^3 - 12\,x^2$

(%o15) $12\,x^2 - 24\,x$

Find the critical points by setting Y' = 0

```
(%i16) CP:solve(Y1=0,x);
```

(%o16) $[x = 0, x = 3]$

All of these are real, so we don't need to exclude any of them.
Substitute these values back into Y to find the y-coordinate
and into Y'' to use the second derivative test.

The makelist command generates a list by
iterating through the existing list. It goes from k=1 to k=2,
which gets both of the solutions. If we had a problem
where solutions 1 and 2 were complex, but 3 and 4 where real,
we could use makelist(EXPRESSION,k,3,4)

```
(%i17) CP;
       makelist(subst(CP[k],Y),k,1,2);
       makelist(subst(CP[k],Y2),k,1,2);
```

(%o17) $[x=0,x=3]$

(%o18) $[2,-25]$

(%o19) $[0,36]$

Since the y''(0) is 0, the second derivative test fails,
we need to use the first derivative test.
Pick something to either side of x=0
and plug it into the first derivative.

```
(%i20) subst(x=-0.1,Y1);
       subst(x=0.1,Y1);
```

(%o20) -0.124

(%o21) -0.116

The function is decreasing on both sides of x=0
so it is neither a maximum nor a minimum

However, f''(3) = 36, which is greater than 0.
There is a relative minimum at (3,-25)

2.2 Problem 3.5.79 (using instructions from 3.6.34)

Define the function and its first two derivatives

```
(%i22) Y:(x-2)/(x^2-4*x+3),factor;
```

(%o22) $$\frac{x-2}{(x-3)(x-1)}$$

The vertical asymptotes are at x=3 and x=1
The x-intercept is at x=2

Find the y-intercept

(%i23) subst(x=0,Y);

(%o23) $-\frac{2}{3}$

A sign chart for Y indicates whether the graph is above or
below the x-axis.

Y is positive on (1,2) and (3,inf)
Y is negative on (-inf,1) and (2,3)

Horizontal asymptotes are determined by finding the limits
as x approaches infinity or negative infinity

(%i24) limit(Y,x,inf);
limit(Y,x,-inf);

(%o24) 0

(%o25) 0

Because these limits are both 0, the graph is
asymptotic to the x-axis to the far right and far
left of the graph

Critical points are where Y' is zero or undefined

(%i26) Y1:diff(Y,x),factor;
solve(Y1,x);

(%o26) $-\frac{x^2-4x+5}{(x-3)^2(x-1)^2}$

(%o27) $[x=2-\%i,x=\%i+2]$

There are no real values where Y' = 0,
the only places where Y' is undefined are not in the domain of Y
There are no critical points;
there are no relative maximums or minimums.

Y' is always negative, so the function is always decreasing

Concavity is given by Y''

(%i28)
```
Y2:diff(Y,x,2),factor;
InfPts:solve(Y2,x);
```

(%o28) $$\frac{2\left(x-2\right)\left(x^2-4\,x+7\right)}{\left(x-3\right)^3\left(x-1\right)^3}$$

(%o29) $$[x=2-\sqrt{3}\,\%\mathrm{i},x=\sqrt{3}\,\%\mathrm{i}+2,x=2]$$

```
Y'' = 0 gives an inflection point at x=2,
so concavity changes at x=2
We can find the y-value by finding Y(2)
```

(%i30)
```
subst(InfPts[3],Y);
```

(%o30) 0

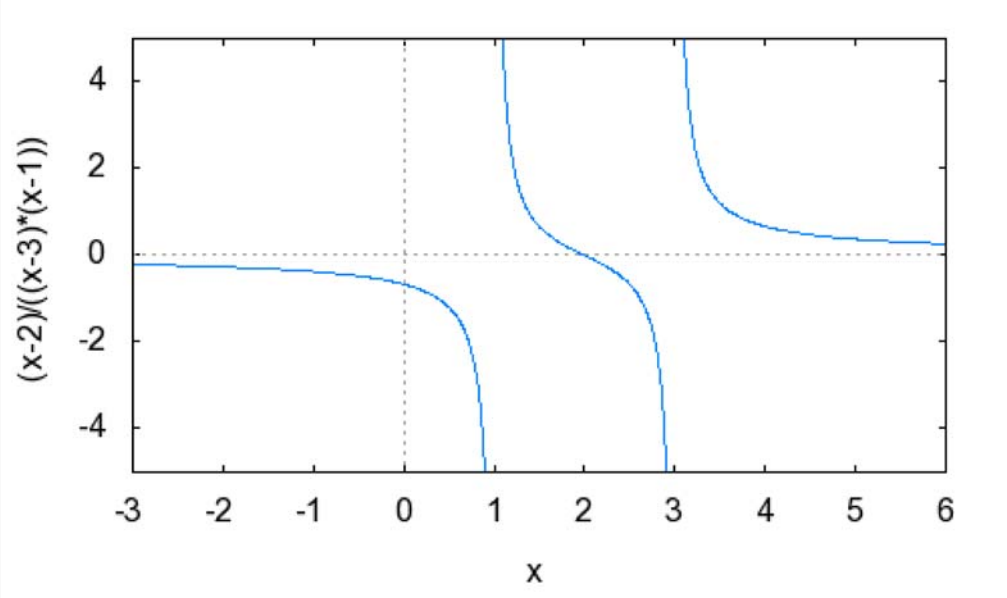
```
There is an inflection point at (2,0);
```

```
Y'' is positive on (1,2) and (3,inf)
  so Y is concave up there
Y'' is negative on (-inf,1) and (2,3)
  so Y is concave down there
```

(%i31)
```
wxplot2d([Y], [x,-3,6],[y,-5,5])$
```

plot2d: some values were clipped.

(%t31)



3 See Word Document