Example Technology Excercise 4 John Smith and Tom Brown

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2 Find the integrals

2.1 Problem 4.4.13

(%i1) integrate(3/x^2-1,x,1,2);

(%o1) $\frac{1}{2}$

2.2 Modified Problem 4.4.25

This one is tricky, so I'm going to label the function to make it easier to refer to later on.

(%i2) Y:x^2-4*x-5;

(%o2) $x^2-4\,x-5$

(%i3) integrate(abs(Y),x,-2,6);

(%o3) $\int_{-2}^{6} \left| x^2-4\,x-5 \right| \mathrm{d}x$

The first attempt to do this failed.
I guess it's too complicated for Maxima
 to handle directly.

Let's factor it so we can see what's going on.

(%i4) factor(x^2-4*x-5);

(%o4) $\left(x-5\right)\left(x+1\right)$

Ahh, we see that it changes signs at x=5 and x=-1
The function is positive on (-inf,-1) and (5,inf)
and negative on (-1,5)

Let's break it into three integrals

(%i5) integrate(Y,x,-2,-1)+integrate(-Y,x,-1,5)+integrate(Y,x,5,6);

(%o5) $\frac{128}{3}$

2.3 Problem 4.5.25

(%i6) integrate(x^2/(1+x^3)^2,x);

(%o6) $-\frac{1}{3\left(x^3+1\right)}$

2.4 Problem 4.6.37

Declare the function

(%i7) Y:tan(x^2);

(%o7) $\tan\left(x^2\right)$

Let's start with the Trapezoid method
The error <= (b-a)^3/(12n^2)*|max Y''|

b-a = 1 - 0 = 1
We're trying to find n

Find the second derivative

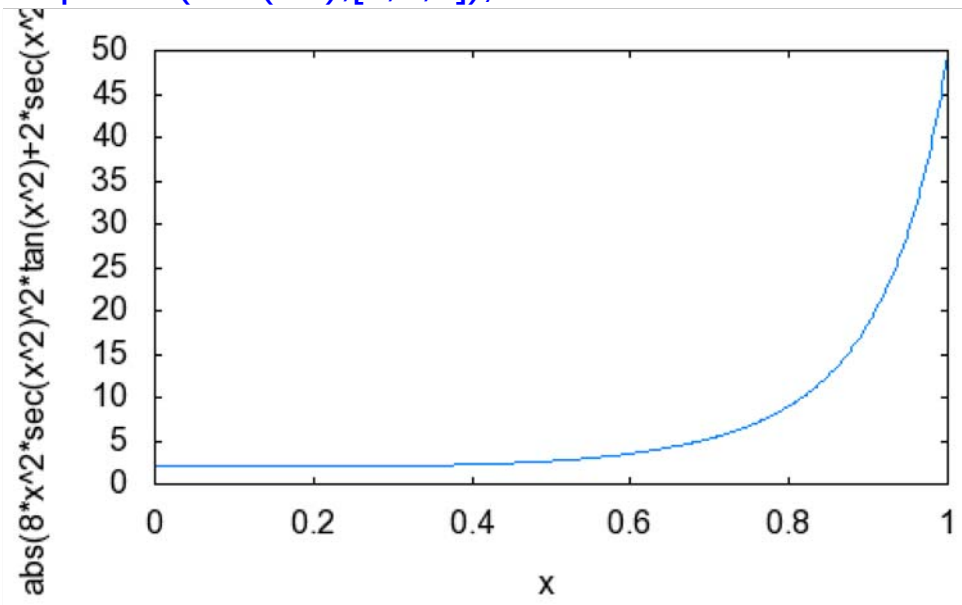
(%i8) Y2:diff(Y,x,2);

(%o8) $8\,x^2\sec\left(x^2\right)^2\tan\left(x^2\right)+2\sec\left(x^2\right)^2$

Find the largest value of the abs(Y'')
on the interval [0,1]

(%i9) wxplot2d(abs(Y2),[x,0,1]);

(%t9)



(%o9)

It appears the largest value happens when x=1, so we could be more precise ...

(%i10) K2:float(abs(subst(x=1,Y2)));

(%o10) 49.53047342153078

We need for the error to be less than 0.00001
Let's set the error equal to that and solve for n
The ratprint: false$ is to suppress some warning messages.

(%i11) ratprint: false$
solve(0.00001 = 1^3/(12*n^2)*K2,n),float;

(%o12) $[n=-642.4593023255814, n=642.4593023255814]$

Since n must be an integer, we can't use 642.4593023255814.
We need to round it up to 643. You always round up in these problems otherwise the error may be too big.

Note: You're not bound to use the true value of |Y''| and it's not always easy to find. You can actually pick ANY value larger than the maximum |Y''|, so you could use 50 instead of 49.53047342153078 if you want to.

Now let's tackle Simpson's method
The error <= (b-a)^5/(180n^4)*|max Y^(4)|

Let's find the fourth derivative

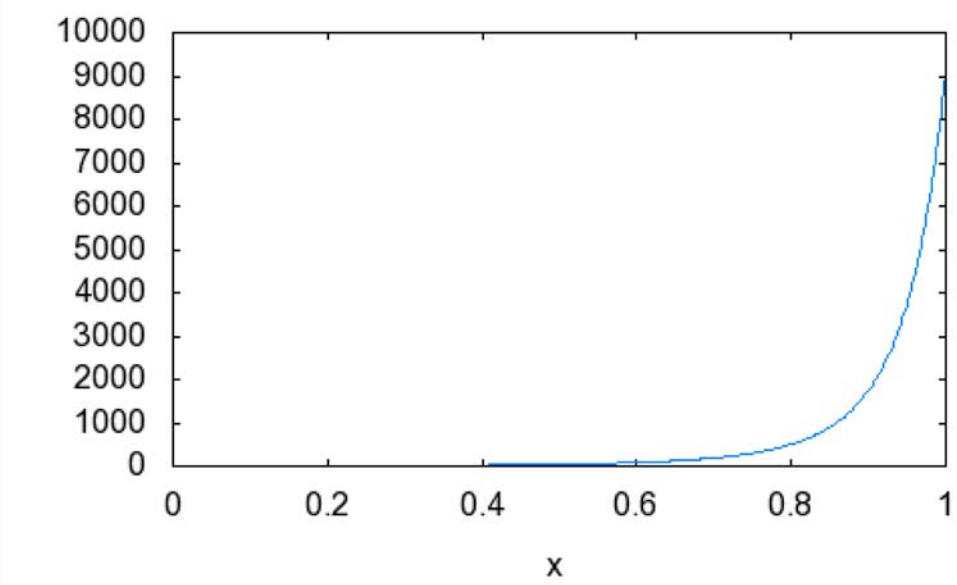
(%i13) Y4:diff(Y,x,4);

(%o13) $128\,x^4\sec\left(x^2\right)^2\tan\left(x^2\right)^3+192\,x^2\sec\left(x^2\right)^2\tan\left(x^2\right)^2+256\,x^4\sec\left(x^2\right)^4\tan\left(x^2\right)+24\sec\left(x^2\right)^2\tan\left(x^2\right)+96\,x^2\sec\left(x^2\right)^4$

And graph it to find the maximum value

(%i14) wxplot2d(abs(Y4),[x,0,1]);

(%t14)



(%o14)

It appears the largest value happens when x=1, so we could be more precise ...

(%i15) K4:float(abs(subst(x=1,Y4)));

(%o15) 9184.473433729336

We need for the error to be less than 0.00001
Let's set the error equal to that and solve for n

(%i16) solve(0.00001 = 1^3/(180*n^4)*K4,n),float;

(%o16) $[n=47.52755102040817\ \%i, n=-47.52755102040816, n=-47.52755102040817\ \%i, n=47.52755102040816]$

If you round the 47.52755102040816 up to the
next even integer, you get n = 48.

Note:
In this problem, both K2 and K4 occurred at one
of the endpoints, so it was easy to see where it
was. If it occurs somewhere in the middle, then
you can use the graph to find a left and right
bound and then use the find_root() on the next
higher ordered derivative. You use either the
third derivative (to maximize the second) or the
fifth derivative to maximize the fourth. For
example, if the graph showed the highest value
of |Y''| was between 0.3 and 0.5, then you could
do this to find K2.

```
find_root(diff(Y,x,3),x,0.3,0.5);
K2:subst(x=%,Y2);
```

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