

# Example Technology Excercise 5 John Smith and Tom Brown

## 1 *Description of region*

The region is bounded by:

```
(%i1) top: 4-0.5*x$
      bottom: 0.2*(x-3)^2+1$
      left: 1$
      right: 4$
```

Suppress the warnings about rational numbers

```
(%i5) ratprint:false$
```

## 2 *Find the area*

```
(%i6) integrate(top-bottom,x,left,right);
(%o6)  $\frac{93}{20}$ 
```

## 3 *Find the volume when the region is rotated about the x-axis*

This uses the washer method

```
(%i7) %pi*integrate(top^2-bottom^2,x,left,right);
(%o7)  $\frac{9393 \pi}{500}$ 
```

## 4 *Find the volume when the region is rotated about the y-axis*

This uses cylindrical shells

```
(%i8) 2*%pi*integrate(x*(top-bottom),x,left,right);
(%o8)  $\frac{219\pi}{10}$ 
```

□ **5 Find the volume when the region is rotated about  $x=-5$**

∟ This uses cylindrical shells

```
(%i9) 2*%pi*integrate((x+5)*(top-bottom),x,left,right);
(%o9)  $\frac{342\pi}{5}$ 
```

□ **6 Find the perimeter of the region**

∟ First the top function

```
(%i10) len_top:integrate(sqrt(1+diff(top,x)^2),x,left,right),float;
(%o10) 3.354101966249685
```

∟ Now the bottom

```
(%i11) len_bottom:integrate(sqrt(1+diff(bottom,x)^2),x,left,right),float;
(%o11) 3.222520797970178
```

∟ The left is easy since it's a vertical line segment

```
(%i12) len_left:subst(x=left,top-bottom);
(%o12) 1.7
```

∟ Similarly, the right is also a line segment

```
(%i13) len_right:subst(x=right,top-bottom);
(%o13) 0.8
```

∟ Now find the perimeter by adding all those together

`(%i14) len_top+len_bottom+len_left+len_right;`  
 `(%o14) 9.076622764219863`

**7 Find the centroid**

Find the mass

`(%i15) M:integrate(top-bottom,x,left,right);`  
 `(%o15)  $\frac{93}{20}$`

Find  $M_y$ , the moment about the  $y$ -axis ( $x=0$ )

`(%i16) My:integrate(x*(top-bottom),x,left,right);`  
 `(%o16)  $\frac{219}{20}$`

Find  $M_x$ , the moment about the  $x$ -axis ( $y=0$ )

`(%i17) Mx:integrate((top+bottom)/2*(top-bottom),x,left,right);`  
 `(%o17)  $\frac{9393}{1000}$`

The centroid is  $M_y/M$  and  $M_x/M$

`(%i18) xbar:My/M;`  
 `ybar:Mx/M;`  
 `(%o18)  $\frac{73}{31}$`   
 `(%o19)  $\frac{101}{50}$`

**8 Find the volume when the region is rotated about  $2x+3y=18$**

The Theorem of Pappus says that the volume is equal to the area of the region times the distance traveled by the centroid.

We need the distance between the centroid and the line.

```
(%i20) R:abs(2*xbar+3*ybar-18)/sqrt(2^2+3^2),float;
(%o20) 2.005330677127788
```

Now the volume, using M as the area from the last part

```
(%i21) 2*pi*R*M,float;
(%o21) 18.64957529728843 pi
```

**9 Find the area of the surface when the top portion is rotated about x-axis**

The main formula is  $2\pi r l$ , where  $r$  is the radius ( $y$  value) and the  $l$  is the arclength parameter

```
(%i22) 2*pi*integrate(top*sqrt(1+diff(top,x)^2),x,left,right);
(%o22) 18.44756081437327 pi
```