

Example Technology Exercise 4

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1 See Word Document

2 Find the integrals

2.1 Problem 4.4.13

```
(%i1) integrate(3/x^2-1,x,1,2);
```

```
(%o1)  $\frac{1}{2}$ 
```

2.2 Modified Problem 4.4.25

This one is tricky, so I'm going to label the function to make it easier to refer to later on.

```
(%i2) Y:x^2-4*x-5;
```

```
(%o2)  $x^2-4x-5$ 
```

```
(%i3) integrate(abs(Y),x,-2,6);
```

```
(%o3)  $\int_{-2}^6 |x^2-4x-5| dx$ 
```

The first attempt to do this failed.
I guess it's too complicated for Maxima to handle directly.

Let's factor it so we can see what's going on.

```
(%i4) factor(x^2-4*x-5);
```

```
(%o4)  $(x-5)(x+1)$ 
```

Ahh, we see that it changes signs at $x=5$ and $x=-1$
The function is positive on $(-\infty, -1)$ and $(5, \infty)$
and negative on $(-1, 5)$

Let's break it into three integrals

```
(%i5) integrate(Y,x,-2,-1)+integrate(-Y,x,-1,5)+integrate(Y,x,5,6);
```

$$(\%05) \frac{128}{3}$$

2.3 Problem 4.5.25

```
(%i6) integrate(x^2/(1+x^3)^2,x);
```

$$(\%06) -\frac{1}{3(x^3+1)}$$

2.4 Problem 4.6.37

Declare the function

```
(%i7) Y:tan(x^2);
```

$$(\%07) \tan(x^2)$$

Let's start with the Trapezoid method
The error $\leq (b-a)^3/(12n^2) \cdot |\max Y''|$

$b-a = 1 - 0 = 1$
We're trying to find n

Find the second derivative

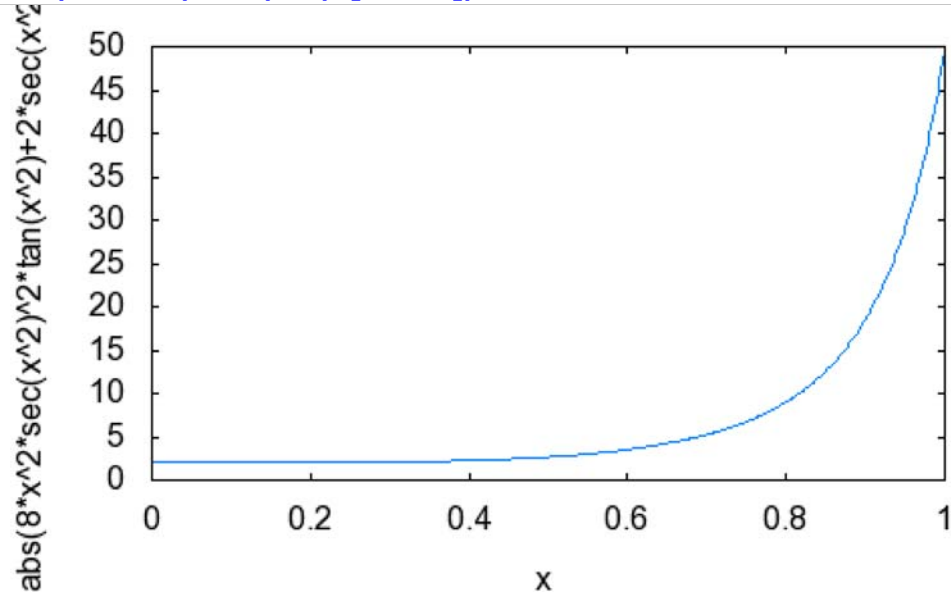
```
(%i8) Y2:diff(Y,x,2);
```

$$(\%08) 8x^2 \sec(x^2)^2 \tan(x^2) + 2 \sec(x^2)^2$$

Find the largest value of the $\text{abs}(Y'')$
on the interval $[0,1]$

```
(%i9) wxplot2d(abs(Y2),[x,0,1]);
```

```
(%t9)
```



```
(%o9)
```

It appears the largest value happens when $x=1$, so we could be more precise ...

```
(%i10) K2:float(abs(subst(x=1,Y2)));
```

```
(%o10) 49.53047342153078
```

We need for the error to be less than 0.00001
Let's set the error equal to that and solve for n
The ratprint: false\$ is to suppress some warning messages.

```
(%i11) ratprint: false$
```

```
solve(0.00001 = 1^3/(12*n^2)*K2,n),float;
```

```
(%o12) [n=-642.4593023255814,n=642.4593023255814]
```

Since n must be an integer, we can't use 642.4593023255814.
We need to round it up to 643. You always round up in these problems otherwise the error may be too big.

Note: You're not bound to use the true value of $|Y''|$ and it's not always easy to find. You can actually pick ANY value larger than the maximum $|Y''|$, so you could use 50 instead of 49.53047342153078 if you want to.

Now let's tackle Simpson's method
The error $\leq (b-a)^5/(180n^4)*|\max Y^{(4)}|$

Let's find the fourth derivative

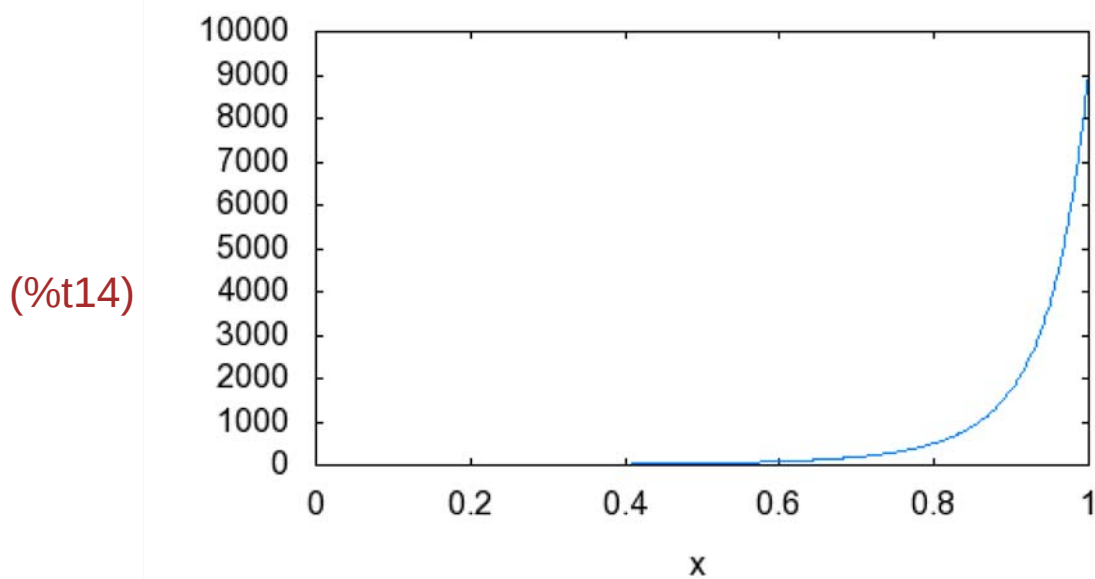
```
(%i13) Y4:diff(Y,x,4);
```

```
(%o13) 
$$128 x^4 \sec(x^2)^2 \tan(x^2)^3 + 192 x^2 \sec(x^2)^2 \tan(x^2)^2 + 256 x^4 \sec(x^2)^4 \tan(x^2) + 24 \sec(x^2)^2 \tan(x^2) + 96 x^2 \sec(x^2)^4$$

```

And graph it to find the maximum value

```
(%i14) wxplot2d(abs(Y4),[x,0,1]);
```



It appears the largest value happens when $x=1$, so we could be more precise ...

```
(%i15) K4:float(abs(subst(x=1,Y4)));
```

```
(%o15) 9184.473433729336
```

We need for the error to be less than 0.00001
Let's set the error equal to that and solve for n

```
(%i16) solve(0.00001 = 1^5/(180*n^4)*K4,n),float;
```

```
(%o16) [n=47.52755102040817 %i,n=-47.52755102040816,n=-47.52755102040817 %i,n=47.52755102040816]
```

If you round the 47.52755102040816 up to the next even integer, you get $n = 48$.

Note:

In this problem, both K2 and K4 occurred at one of the endpoints, so it was easy to see where it was. If it occurs somewhere in the middle, then you can use the graph to find a left and right bound and then use the `find_root()` on the next higher ordered derivative. You use either the third derivative (to maximize the second) or the fifth derivative to maximize the fourth. For example, if the graph showed the highest value of $|Y''|$ was between 0.3 and 0.5, then you could do this to find K2.

```
find_root(diff(Y,x,3),x,0.3,0.5);  
K2:subst(x=%,Y2);
```

3 See Word Document