Skills Needed for Success in Calc 1

There is much apprehension from students taking Calculus. It seems that for many people, "Calculus" is synonymous with "difficult." However, any teacher of Calculus will tell you that the reason that students are not successful in Calculus is not because of the Calculus, it's because their algebra and trigonometry skills are weak. You see, Calculus is really just one additional step beyond algebra and trig. Calculus is algebra and trigonometry with limits and limits aren't really that hard once you figure them out. There is often only one step in the problem that actually involves calculus, the rest is simplifying using algebra and trigonometry. That's why it is crucial that you have a good background in those subjects to be successful in calculus.

More good news about calculus is that we live in the real world, we don't deal with imaginary numbers. In Calculus 1, there is only limited exposure to logarithmic and exponential functions.

The purpose of this document is to help identify some of those areas where you will need good algebra and trigonometry skills so that your calculus experience can be successful, pleasant, and rewarding.

After reading this document, you may think there were a lot of things you had to learn in either College Algebra or Trigonometry that were a waste of your time. This document addresses what you'll need for Calculus 1. There are other things you'll need from those classes in Calculus 2, Calculus 3, and Differential Equations. In addition, the proofs from Trigonometry are vital for developing the thought and reasoning process that allow you take complex problems and break them up into smaller steps. Those proofs also help you learn the trigonometric identities. It is also important to realize that College Algebra and Trigonometry help prepare you for other classes besides Calculus.

Algebra Skills Needed

Factoring

You need to be able to factor expressions and equations like it was second nature to you. Factoring is usually preferred to expanding and can greatly simplify the amount of time it takes to work a problem.

Many of the problems in calculus will involve finding the roots of a function and for the most part that means factoring. Don't just concentrate on polynomial factoring, either; you need to be able to factor expressions with rational exponents.

Here is an example of factoring out the greatest common factor, which is involves taking the smallest exponent on all of the common terms.

$$5x^2(2x-3)^{1/3}(3x+2)^{1/2} + 8x(2x-3)^{-2/3}(3x+2)^{3/2}$$
$$x(2x-3)^{-2/3}(3x+2)^{1/2}\left[5x(2x-3) + 8(3x+2)\right]$$
$$x(2x-3)^{-2/3}(3x+2)^{1/2}(10x^2+9x+16)$$

Know how to recognize and factor the special patterns of the difference of two squares, the difference of two cubes, and the sum of two cubes. Know that the sum of two squares usually doesn't factor in the real world.

- Difference of two squares: $x^2 - y^2 = (x-y)(x+y)$
- Sum / difference of two cubes: $x^3 \pm y^3 = (x \pm y)(x^2 \mp xy + y^2)$

Completing the Square

Another task that you will be called on to perform occasionally is completing the square. You need to be able to do this with both an equation and an expression.

Completing the Square with an Equation

With an equation, you can add the same value to both sides of the equation. Here's an example of completing the square to analyze a conic section.

$$x^2 + 3y^2 + 2x - 12y = 3$$
$$(x^2 + 2x) + 3(y^2 - 4y) = 3$$
$$(x^2 + 2x + 1) + 3(y^2 - 4y + 4) = 3 + 1 + 12$$
$$(x+1)^2 + 3(y-2)^2 = 16$$

Completing the Square with an Expression

With an expression, you do not have an equal sign so you cannot do the same thing to both sides. In other cases, you have an equal sign, but you want to keep the $f(x)$ by iteself. In these cases, you need to add and subtract the same value, which is also known as adding zero.

$$
\begin{aligned}
f(x) &= 5x - x^2 \\
f(x) &= -(x^2 - 5x) \\
f(x) &= -\left(x^2 - 5x + \tfrac{25}{4}\right) + \tfrac{25}{4} \\
f(x) &= -\left(x - \tfrac{5}{2}\right)^2 + \tfrac{25}{4}
\end{aligned}
$$

Simplifying Expressions

Much of your time in this course will be spent simplifying the results of an expression that you obtained. Know how to combine similar or like terms and know the properties of exponents like adding exponents when multiplying factors that have the same base or multiplying exponents when raising to a power.

Using Your Calculator

This may sound like a given condition by the time you get to calculus, but you need to be able to graph functions and get a proper viewing window. You should be able to use the Calc menu on your calculator to find roots, minimums, maximums, and intersections. You should also know how to use the table mode on your calculator. You should also know how to change the mode on your calculator and leave it in Radian mode for most of this course.

Formula Manipulation

You need to be able to work with formulas as well as have a good recall of basic geometry formulas for area and volume for common figures. There are geometric formulas on the inside front cover of your text as a resource.

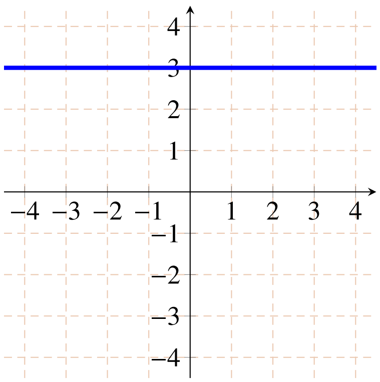
Formula manipulation is much more than just memorizing formulas and plugging the values into them, however. You will need to solve for different variables and you will need to combine formulas together to come up with new formulas.

Example: A right circular cylinder has a volume of $10\,\text{cm}^3$ and its height is twice its circumference. Find the radius and height of the cylinder. The formula for the volume of any cylinder with parallel bases is $V = Bh$, where B is the area of the base. Since the base is a circle, the area of the base is $B = \pi r^2$. The circumference of a circle is $C = 2\pi r$ and in this cylinder, the height is twice the circumference, so the height is $h = 2C = 2(2\pi r) = 4\pi r$. The volume becomes $V = \left(\pi r^2\right)(4\pi r) = 4\pi^2 r^3$. Since we know that the volume is $V = 10$, we get $10 = 4\pi^2 r^3$. Solving that for r gives $r^3 = \frac{5}{2\pi^2}$ or $r = \sqrt[3]{\frac{5}{2\pi^2}}$ cm. The height is $h = 4\pi r = 4\pi\sqrt[3]{\frac{5}{2\pi^2}}$ cm, but that simplifies to be $h = 2\sqrt[3]{20\pi}$ cm.

Basic Functions and Transformations

Algebra (and calculus) can be simplified if you understand that there are basic functions and that many of the other functions are transformations of those basic building blocks. You should be able to sketch the graph and know the domain and range of the basic functions upon sight.

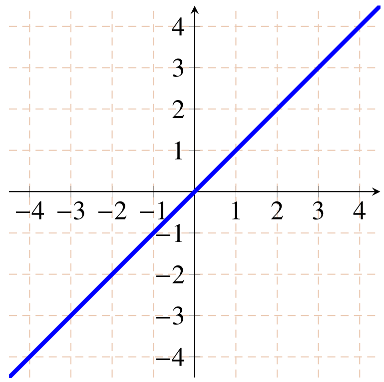
You should also be able to recognize and apply transformations to the basic functions. Consider $y = 2 - (x - 3)^2$. You should be able to recognize that the basic shape is the quadratic function $y = x^2$. To that basic shape, you have reflected it about the x-axis $y = -x^2$, shifted it up two units $y = 2 - x^2$, and shifted it right three units $y = 2 - (x - 3)^2$.



Constant $y = c$

Domain: $(-\infty, +\infty)$

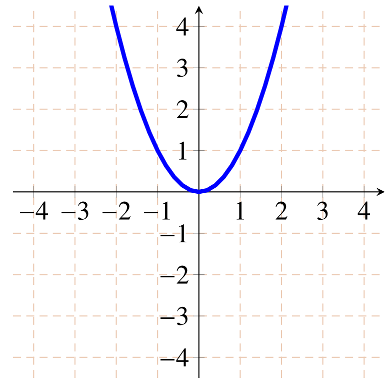
Range: $\{c\}$



Linear $y = x$

Domain: $(-\infty, +\infty)$

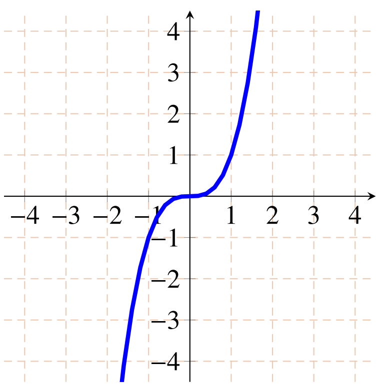
Range: $(-\infty, +\infty)$



Quadratic $y = x^2$

Domain: $(-\infty, +\infty)$

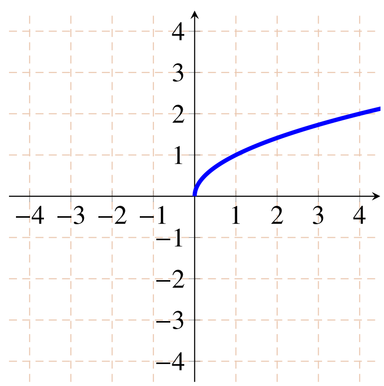
Range: $[0, +\infty)$



Cubic $y = x^3$

Domain: $(-\infty, +\infty)$

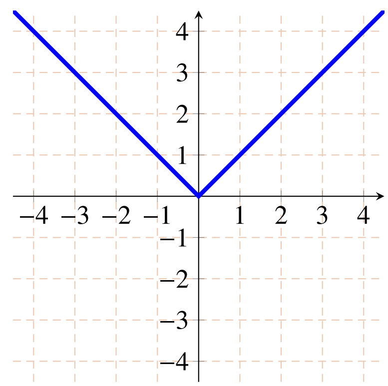
Range: $(-\infty, +\infty)$



Square Root $y = \sqrt{x}$

Domain: $(-\infty, +\infty)$

Range: $[0, +\infty)$



Absolute Value $y = |x|$

Domain: $(-\infty, +\infty)$

Range: $[0, +\infty)$

Trigonometry Skills Needed

There were a lot of formulas in trigonometry. Your trigonometry teacher might have let you use a note card during exams so that you didn't have to memorize them. That will not be the case in calculus. You are expected to have memorized many of the basic values and formulas from your trigonometry class.

While you need to have most of the basic formulas and definitions memorized, other things can be derived by understanding the relationships between the trigonometric functions and the different quadrants. The list of things you need to know isn't as bad as it seems, this document will help you understand which ones are super-important.

Radian Measure

Every problem in calculus is worked in radians. If a problem happens to involve degrees, we convert it to radians, work it out, and then convert it back to degrees if needed. The conversion is formula is 180 deg = π rad. To go from radians to degrees, you multiply by $\frac{180}{\pi}$ and to go from degrees to radians, you multiply by $\frac{\pi}{180}$.

Stop thinking in degrees and start thinking in radians. If the teacher asks where the sine is one-half, do not say 30°. Do not even let 30° be the first thing that comes to your mind. Your first thought and response should be $\frac{\pi}{6}$. There are technically other values like $\frac{5\pi}{6}$, $\frac{13\pi}{6}$, etc., but your first thought should be of the reference angle in the first quadrant and that's often all the instructor is looking for.

Special Angles

Memorize the values of the three trigonometric functions for the special angles in the first quadrant! Yes, they can be derived, but you are going to use them so often as a portion of a much larger problem that you can't afford to take the time to derive them every time you need them.

Special Angles and their Trigonometric Values

θ	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$
$\sin\theta$	0	$1/2$	$\sqrt{2}/2$	$\sqrt{3}/2$	1
$\cos\theta$	1	$\sqrt{3}/2$	$\sqrt{2}/2$	$1/2$	0
$\tan\theta$	0	$1/\sqrt{3}$	1	$\sqrt{3}$	undefined

In this class, you will be expected to give exact answers in most cases. That means writing $1+\sqrt{2}$ instead of 2.414 or $\frac{17}{3}\pi$ and not 17.802. The good news is that you will not usually have to rationalize your denominators and writing $\frac{1}{\sqrt{2}}$ is okay.

Formulas

These formulas are used frequently enough that you should have them memorized or quickly be able to derive them:

- Pythagorean relationships: $\cos^2\theta+\sin^2\theta=1$, $1+\tan^2\theta=\sec^2\theta$, and $1+\cot^2\theta=\csc^2\theta$

- The sum and difference formulas: $\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$ and $\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$
- Double angle formulas: $\sin 2\theta = 2 \sin\theta \cos\theta$, $\cos 2\theta = \cos^2\theta - \sin^2\theta$, $\cos 2\theta = 2\cos^2\theta - 1$, and $\cos 2\theta = 1 - 2\sin^2\theta$
- Half angle formulas: $\cos^2\theta = \frac{1+\cos 2\theta}{2}$ and $\sin^2\theta = \frac{1-\cos 2\theta}{2}$. *These are not how they were written in Trigonometry class, but are how they will be used in calculus.*

Missing from this list are most of the formulas for the tangent function. That's because they can usually be derived from the sine and cosine formulas. The law of sines and law of cosines isn't used much, if ever, in calculus. The product to sum formulas are used so rarely that it's easier to just look them up when they're needed.

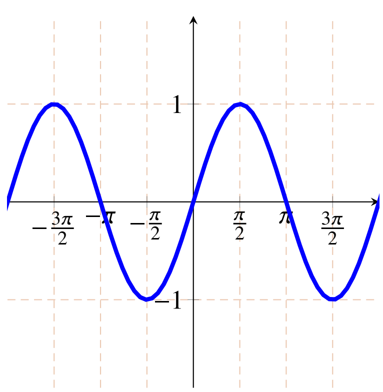
Quadrants, Signs, and Reference Angles

You do not need to memorize the values of the trig functions for the special angles outside the first quadrant as long as you understand the concept of reference angles and the signs of the functions in the various quadrants. For example, let's say you were to find the exact value of $\sin\frac{5\pi}{3}$. The reference angle is $\frac{\pi}{3}$ and you have memorized that $\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$. $\frac{5\pi}{3}$ is in quadrant IV and the sine function is negative in quadrant IV, so the answer is $\sin\frac{5\pi}{3} = -\sin\frac{\pi}{3} = -\frac{\sqrt{3}}{2}$.

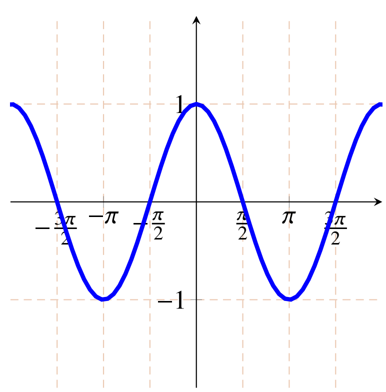
In trigonometry class, you had to take problems like and simplify them. You were probably expected to use the sum of two angles formula for sine and go . In calculus, you could do that, but it takes way too long. What you're expected to know here is that if is in the first quadrant, then is in the third quadrant and in the third quadrant, the sine is the opposite of what it is in the first quadrant, so .

Basic Functions and Transformations

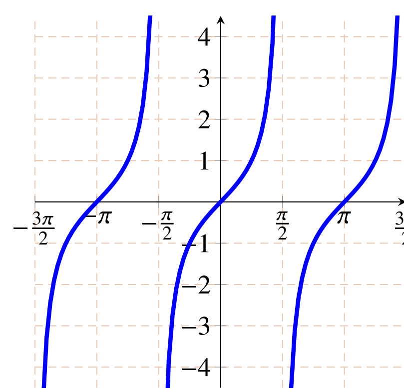
You should be able to sketch the basic trigonometric functions and be able to apply transformations to them. For example, consider the function $y = 3\sin(2x-1)+5$. You should be able to pick out that the basic graph is the sine function from $y = \sin x$, that the amplitude is 3 because of $y = 3\sin x$, the period is $\frac{2\pi}{2} = \pi$ from $y = 3\sin(2x)$, the phase shift is $\frac{1}{2}$ unit to the right from $y = 3\sin\left[2\left(x-\frac{1}{2}\right)\right]$, and the entire graph has been shift up five units from $y = 3\sin\left[2\left(x-\frac{1}{2}\right)\right]+5$.



Sine $y = \sin x$
Domain: $(-\infty, +\infty)$
Range: $[-1, 1]$



Cosine $y = \cos x$
Domain: $(-\infty, +\infty)$
Range: $[-1, 1]$



Tangent $y = \tan x$
Domain: $x \neq \frac{\pi}{2} \pm k\pi$
Range: $(-\infty, +\infty)$

Geometry Skills Needed

You will be expected to know some of the basic formulas for area and volume from plane geometry. Listed below is a minimum set of formulas that should be memorized. There are other formulas that will be used, like the area of a sector, but they will be covered in class when needed.

- Circles
  - Circumference of a circle $C = 2\pi r$
  - Area of a circle $A = \pi r^2$
- Areas are the average length of the bases times the height: $A = \frac{1}{2}(b_1 + b_2)h$
  - Trapezoid $A = \frac{1}{2}(b_1 + b_2)h$
  - Parallelogram (the bases have the same length $b = b_1 = b_2$) $A = bh$
  - Triangle (the vertex has length $b_2 = 0$) $A = \frac{1}{2}bh$
- Lateral surface area is the average perimeter of the bases times the slant height l: $LSA = \frac{1}{2}(p_1 + p_2)\, l$
  - Prism or cylinder (bases are congruent so $p = p_1 = p_2$): $LSA = pl$
  - Pyramid or cone (1/2 the area of a prism or cylinder because $p_2 = 0$): $LSA = \frac{1}{2}pl$
- Volumes
  - Prism or cylinder (area of the base times the height) $V = Ah$
  - Pyramid or cone (1/3 the volume of a prism or cylinder) $V = \frac{1}{3}Ah$

In geometry, you were probably forced to memorize formulas for things like the volume of a right circular cylinder $V = \pi r^2 h$. It's great if you know these, but it's better if you realize a right circular cylinder is a cylinder with a circle for a base. Then you take the area of the circular base $A = \pi r^2$ and multiply it by the height h to get $V = Ah = \pi r^2 h$. This really cuts down on the number of formulas needed and it helps you start thinking in general terms and seeing relationships between objects rather than treating each problem in isolation.

Prisms and cylinders are treated the same, the technical difference being whether the base is a polyhedron (straight edges) or a curve. Cylinders do not have to be circular. Pyramids and cones are prisms and cylinders that go to a point at one end.

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