Math 160 - Projects

This course has some projects that require you to go beyond the level of material covered in the book or use outside sources to acquire data.

Although you may work the problems individually, the projects are designed to be group projects. They require extra work and you can benefit from the group. The exams will often see questions similar to those on the projects and so having other people verify the correctness of the problems before you take the exam can be beneficial.

Groups may have up to 3 people in them (except for the project 3A, which must be worked individually). You will be able to select your own groups for the projects and you may with each unit. Most submissions in Canvas ask you to identify who you worked with on the projects, even if you worked alone.

Some of these projects are very similar to problems that will appear on your exams. For that reason, it is important that you complete and understand them before taking your exam, even if they are not due until after the exam.

In the projects, there are blanks _____ to indicate items that need answered. Projects will be turned in electronically and so make sure you answer those questions in the projects before you submit them.

The assignments in Canvas contain links to templates and forms for completing your projects. Some of those templates have one page for the work and another page for answers; others have work at the top and answers at the bottom. Make sure that you answer the questions.

Completed projects must be submitted using Canvas. Although you may work in groups, each person must individually submit their answers.

Project 4A: Baseball (Ch 4)

A dominance relation matrix is a pre-cursor to transition matrices in chapter 9. You have a square matrix with the same labels for rows and columns. The value in any row and column represents the number of times the row player beat the column player.

For example, in the dominance relation shown in the table, Baltimore beat Boston 10 times, Boston beat New York 8 times, and Tampa Bay beat Toronto 9 times. The 0's on the main diagonal are because the teams never beat themselves (at least literally).

One use of the dominance relation matrix is in computing indirect winners. The idea being that if team A beats team B and team B beats team C, then team A is indirectly better than team C since they beat someone who beat team C.

2017 American League East					
	BAL	BOS	NYY	TBR	TOR
BAL	0	10	7	8	12
BOS	9	0	8	11	13
NYY	12	11	0	12	9
TBR	11	8	7	0	9
TOR	7	6	10	10	0

Create a Google Spreadsheet for this project. Share it with the group members and the instructor. Create a dominance relation matrix for the **2019 National League Central Division** of Major League Baseball and call it matrix A.

You can do this quickly by using the information at baseball-reference.com. Find a team, and then under the "Schedule & Results" for each team, they have a "Team Win/Loss Splits" that contains an "Opponent" column that lists how many times the team won or lost against the opponent (you may need to expand the column to see all the teams). When gathering information, limit yourself to the National League Central Division teams.

For example, the 2019 Chicago Cubs information is at https://www.baseball-reference.com/teams/CHC/2019-schedule-scores.shtml

In an effort to predict the rankings for the 2023 season, a bookie awards points as follows: 6 points for each time the row team directly beats the column team, 3 points for each time the row team beat a team who beat the column team, and 1 point for each time the row team beat a team who beat a team who beat the column team.

_____ Give a formula involving matrix A that computes the total points for each team.

_____ What is the number of points the Brewers directly received from the Cubs?

_____ What is the total number of points the Cubs received from the Reds?

_____ List the teams and their overall scores. Who is the best team (scored most points)?

Project 4B: Networking (Ch 4)

The incidence matrix is similar to the dominance relation matrix, except that the value in a row and column represent the number of direct routes from the row to the column.

Although they don't service Central Illinois, Cox is a major cable company. They have a map of their business network available. You may want to open the version saved in Canvas if you have trouble viewing this one: https://ww2.cox.com/wcm/en/business/datasheet/national-ip-backbone-map.pdf

Create a Google Spreadsheet for this project and share it with the group members.

Create an incidence matrix for the Cox business network, assuming that all traffic is bi-directional, between the regional data centers and call it matrix A. Ignore any local market backbone nodes. List all the cities alphabetically. A hop occurs every time a data packet goes from one center to another along a network segment.

Be careful: You can get from Atlanta to Dallas 1) directly or by going through 2) Pensacola, 3) Macon, or 4) Gainesville so you would enter a 4 for that route. You would not include Atlanta to Baton Rogue to Dallas since Baton Rouge is a regional data center itself. The sum of each row should equal the number of paths leaving the regional data center except for weirdness like Baton Rouge or Kansas City that have loops going back to themselves without hitting going to another regional data center.

_____ The minimum number of hops needed to get from Omaha to Orange County

_____ The regional data centers that are the furthest apart network-wise. That is, which sites require the most segments to get between them?

_____ The minimum number of hops needed to guarantee a packet can travel from any data center to any other data center

_____ The matrix calculation used to find the previous two answers. Part of designing a good network is building in redundancy. If a regional center goes offline, the traffic needs to be able to get from it source to its destination, although it may need additional hops to do so.

_____ The minimum number of hops needed to guarantee a packet can travel from any data to any other data center if Kansas City were to go offline.

Project 8: Decision Theory (Ch 8)

Tom runs a computer store.

He can purchase 10 computers from Quanta for $850 each, 30 computers from Compal for $800 each, or 50 computers from Wistron for $750 each. He can order from more than one manufacturer, but only one order from each.

During the month, Tom sells the computers for $1100 each. Any unsold computers at the end of the month are sold (for sure) for $600. Tom estimate a loss of goodwill of $100 for each customer which comes into the store, but is unable to purchase a computer.

He estimates the monthly demand will be 15, 30, 45, or 60 computers with probabilities of 0.1, 0.4, 0.3, and 0.2 respectively.

_____ Create a payoff table with the five actions (remember that you can combine purchases from more than one dealer and some plans don't make any sense when the demand is considered) and four states of nature (demand)

_____ Create the opportunistic loss (regret) table.

_____ For each decision criteria (expected value, maximax, maximin, minimax), find the payoff or loss for each action and the best action.

Payoff Table

	Buy 10	Buy 30	Buy 40	Buy 50	Buy 60
Demand 15					
Demand 30					
Demand 45					
Demand 60					

Opportunistic Loss Table

	Buy 10	Buy 30	Buy 40	Buy 50	Buy 60
Demand 15					
Demand 30					
Demand 45					
Demand 60					

Actions

Criterion	Buy 10	Buy 30	Buy 40	Buy 50	Buy 60	Best Action
Exp. Value						
Maximax						
Maximin						
Minimax						

Project 9A: Airline Loyalty (Ch 9)

The Central Illinois Regional Airport (CIRA) in Bloomington-Normal is served by American Airlines, Delta, and Allegiant. Of these, only American Airlines and Delta have daily flight service, so we will consider only those two airlines in this analysis.

Trip Advisor has airline recommendations and ratings at https://www.tripadvisor.com/Airlines.

Whether or not a person would recommend an airline is implicitly determined from the overall rating for that airline. For example, if an airline has a rating of 3.5 out of 5 stars, we will take that to mean that 3.5/5.0 = 0.70 or 70% of people would recommend that airline.

Assume that if a customer would recommend an airline to someone that they will fly that airline again the next time they fly. If they would not recommend an airline to someone, then they will fly a competitor airline the next time they fly. Cost or destination is not a factor in choosing an airline.

John has recently started a new job in Bloomington that will require him to fly often. He has no previous experience flying, so he looks at the number of ratings each airline has received on Trip Advisor and randomly chooses an airline based on the relative frequency of ratings.

To help you understand, pretend that American has received 250 ratings and Delta has received 175 ratings, then there are 425 ratings all together. There is a 250/425 chance John will pick American and a 175/425 chance he will pick Delta for the initial flight.

Do not round any of the values until you are done with the calculations (use 250/425, not 0.59). Give at least four decimals in your answers.

_____   Draw a transition diagram and give the transition matrix for John's flights out of CIRA.

_____   What is the probability that John's initial flight will be with Delta?

_____   What is the probability that John's fourth flight will be with American Airlines?

_____   What is the long term probability that John will fly with Delta?

Project 9B: Maze Movement (Ch 9)

A 5×5 grid is laid out as shown in the figure. Each node is labeled using a letter A-E for the row and a number 1-5 for the column. The arrows indicate the direction of movement between nodes.

You begin at a node and then, with each transition, either move to an connected node if you are at a blue circle or remain where you are if you are at a red square.

When moving, you randomly select, with equal probability, a node from any of the connected nodes. You may not remain in the current node if you have the possibility of moving. The probability of moving is determined by the number of adjacent nodes.

For example, if you are at node C3, you can move to nodes B4, C4, or D4, so each would have a probability of 1/3. But if you were at node C4, then you could move to nodes C3 or C5, so the probability of each would be 1/2.

Movement continues until reaching a node with no exit (a square with a red background).

Pay attention to the arrows. You can move from node A1 to node A2 but not the other way around.

A1 → A2   A3 ← A4 ↔ A5

B1 ↔ B2 → B3 ← B4   B5

C1 → C2   C3 ↔ C4 ↔ C5

D1 ↔ D2 ↔ D3 ↔ D4 ↔ D5

E1   E2 → E3   E4 ← E5
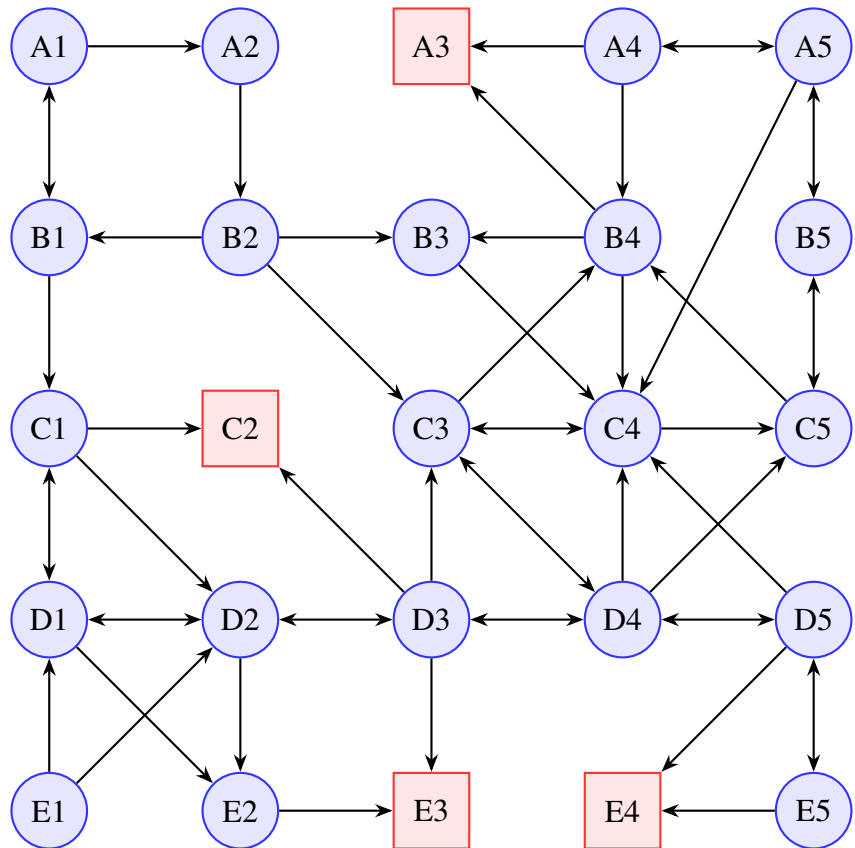
_____ Create a transition matrix in standard form.

_____ Find the fundamental matrix F.

_____ If you start at node D2, how many times can you expect to spend in node C3?

_____ If you start at node C1, how many transitions can be expected before ending?

_____ If you start at node C4, how many transitions can be expected before ending?

_____ If you start at node A2, what is the probability of eventually ending at node E3?

_____ If the object is to make as many transitions as possible before reaching an absorbing node, at which node(s) should you start?

_____ If the object is to end at node A3, in which non-adjacent transient node(s) should you start to maximize your chance?

Project 10: Competing Stores (Ch 10)

Rick and Corissa own the only two grocery stores in town. This means that a sale for Rick is a loss for Corissa and vice versa.

Each week, they each run a special on exactly one type of food in an effort to draw business into their store. The matrix showing the choices and the gain in sales for Rick's store is provided.

	Cereal	Dairy	Health	Meats
Baking	2	−1	−3	0
Fruits	−3	4	1	−2
Pasta	−1	0	−2	2
Seafood	3	−2	2	−3

Rick, the row player, runs specials on baking, fruits, pasta, or seafood.

Corissa, the column player, runs specials on cereal, dairy, health, and meats.

Answer the following questions. Give all answers as fractions!

_____ If Rick and Corissa each randomly select a food type to put on sale, what are the strategies and what is the value of the game for Rick?

_____ What are the optimal strategies for Rick and Corissa? What is the value of the game for Rick under those strategies?

_____ Find the expected payoff values for each of Rick's actions if Corissa plays her optimal strategies. Find the expected loss values for each of Corissa's actions if Rick plays his optimal strategies.

Rick conducts some espionage and finds out that Corissa is going to spin the spinner from the game Life (the spinner has the numbers from 1 to 10 and each is equally likely).

He learns that Corissa will place cereal on sale if the spinner lands on a 1, dairy if it lands on a 2 or 3, health if it lands on 4, 5, or 6, and meats if it lands on a 7, 8, 9, or 10.

_____ If Corissa plays uses this strategy, what is the expected value of each action for Rick?

_____ What should Rick's *a priori* strategy be using the expected value criterion? *a priori* means with prior knowledge – that is, if Rick knows what Corissa is going to do, then what should Rick do?

_____ What should Corissa's strategy really be (not what she said) if she knew he had spies and the intent was to trick Rick into playing a particular strategy?

Decision Theory: Find the best actions for Rick under the expected value, maximax, maximin, and minimax criteria. Use Rick's optimal strategy when finding the expected values.

Actions					
Criterion	Baking	Fruits	Pasta	Seafood	Best Action
Exp. Value					
Maximax					
Maximin					
Minimax					

Project 3A: Planning for Retirement (Ch 3)

This project is to designed to help you get a sense of what is needed to retire. When estimating costs, remember that inflation will occur and things will be more expensive than they are now, but that by the that time , you are likely to have most major purchases like a house or car already purchased.

All ages should be in years, and for simplicity, we will assume that all major events (start saving, retiring, and dying) occur on your birthday so that we don't have to mess with partial years. For purposes of this project, you must not plan on retiring for at least 15 years. Round all dollar amounts to the nearest cent. Assume that all money is invested and earns interest at an annual nominal rate of 9%, compounded monthly.

This is an individual project. You may help each other, but the numbers need to be yours.

_____ Age, in years, that you will be on your next birthday

_____ Age, in years, when you plan on retiring and _____ years until retirement

_____ Number of years that you expect to live and _____ years of retirement

_____ Amount, in dollars, that you anticipate needing each month during your retirement

_____ Amount of money that you need to have saved the day you retired in order to fund your retirement

_____ Amount of money that you will need to save each month until you retire in order for you to reach the amount needed to retire

After 10 years, a trust fund established upon the death of your grandparents gives you a one-time payment of $150,000. You spend $50,000 of it to pay off some college and credit card debts and put the rest of it into your retirement fund and continue making your regular deposits until you retire.

_____ Amount of money you will have after 10 years of regular payments into your retirement fund

_____ Amount of money the remaining $100,000 from the trust fund will grow to before you retire

_____ Amount of money you'll be able to spend each month during retirement when you include the investment results from the trust fund

Project 3B: Buying a Home (Ch 3)

The purpose of this project is to help you understand purchasing a home. You will make a 20% down payment to avoid escrow costs and finance the balance over 30 years with a fixed loan rate of 3.27%. Payments and compounding will occur monthly. Round all dollar amounts to the nearest cent.

Find a home in the Richland district that is between $100,000 and $250,000 by visiting https://www.realtor. com, https://www.zillow.com or some other online real-estate system.

_____   Address of home

_____   Asking price of home

_____   Amount financed after 20% down payment

_____   Monthly payment needed to finance home

_____   Total cost to repay loan if regular payments are made for the full 30 years (the loan does not include the down payment)

_____   Amount of interest paid if the loan goes full term

For the next two questions, assume that ten years of regular payments have been made.

_____   Amount still owed on the house

_____   Equity in home if the house is appraised at 8% more than you originally paid for it

For the next two questions, assume that from the beginning of the loan, you have paid an extra $300 per month. This section has nothing to do with the 10 years in the previous two questions.

_____   The number of months it will take to pay off the loan when you pay an extra $300 per month

_____   The amount of interest you save by paying an extra $300 per month