Pivoting to perform Gauss-Jordan Reduction

Solve the system of linear equations:

\[ \begin{align*}
    x + 2y + 3z &= 7 \\
    -2x + 3y - z &= 5 \\
    -x + 2y + 3z &= -1
\end{align*} \]

Gaussian Reduction places a matrix into row-echelon form, but requires back-substitution. Gauss-Jordan Reduction places the matrix into reduced row-echelon form and does not require back-substitution.

\[
\begin{bmatrix}
    1 & 2 & 3 & | & 7 \\
    0 & 7 & 5 & | & 13 \\
    0 & 0 & 1 & | & 2
\end{bmatrix} \sim \begin{bmatrix}
    1 & 2 & 3 & | & 7 \\
    0 & 7 & 5 & | & 13 \\
    0 & 0 & 1 & | & 2
\end{bmatrix}
\]

\( \checkmark \) is sum of each row.
Select any non-zero element on left side for the pivot element.

\[
\begin{bmatrix}
    1 & 2 & 3 & | & 7 \\
    0 & 7 & 5 & | & 13 \\
    0 & 0 & 1 & | & 2
\end{bmatrix} \sim \begin{bmatrix}
    1 & 2 & 0 & | & 4 \\
    0 & 1 & 0 & | & 3 \\
    0 & 0 & 1 & | & 2
\end{bmatrix}
\]

Select new pivot element.
You may only pivot once in any row or column.

\[
\begin{bmatrix}
    1 & 2 & 0 & | & 4 \\
    0 & 1 & 0 & | & 3 \\
    0 & 0 & 1 & | & 2
\end{bmatrix} \sim \begin{bmatrix}
    1 & 0 & 0 & | & 0 \\
    0 & 1 & 0 & | & 2 \\
    0 & 0 & 1 & | & 2
\end{bmatrix}
\]

Select new pivot element.
Pivot on a 1 when possible.
Only once per row or column.
Since row 3 already has a 0 in the pivot column, we don't need to clear it.

\[
\begin{bmatrix}
    1 & 0 & 0 & | & 0 \\
    0 & 1 & 0 & | & 2 \\
    0 & 0 & 1 & | & 2
\end{bmatrix}
\]

Rewrite pivot row.
Clear pivot column.
Previously cleared columns will remain cleared.
Rewrite any row with a 0 already in pivot column.

How to perform the actual pivot

\[
\begin{bmatrix}
    1 & 2 & 3 & | & 7 \\
    -2 & 3 & -1 & | & 5 \\
    -1 & -2 & 3 & | & -1
\end{bmatrix}
\]

Repeat process for each element that is unknown in next matrix.

Take the difference of the products of the corners of the box formed by the pivot element and the element to be replaced. Always take the diagonal with the pivot element minus the diagonal without the pivot element.

New values for Row 2

\[ y: 1(3) - (-2)(2) = 3 + 4 = 7 \]
\[ z: 1(-1) - (-2)(3) = -1 + 6 = 5 \]
\[ \text{rhs: } 1(5) - (-2)(7) = 5 + 14 = 19 \]
\[ \checkmark: 1(5) - (-2)(13) = 5 + 26 = 31 \]

New values for Row 3

\[ y: 1(-2) - (-1)(2) = -2 + 2 = 0 \]
\[ z: 1(3) - (-1)(3) = 3 + 3 = 6 \]
\[ \text{rhs: } 1(-1) - (-1)(7) = -1 + 7 = 6 \]
\[ \checkmark: 1(-1) - (-1)(13) = -1 + 13 = 12 \]

Pivot

New values for Row 3

\[ y: 1(3) - (-2)(2) = 3 + 4 = 7 \]
\[ z: 1(-1) - (-2)(3) = -1 + 6 = 5 \]
\[ \text{rhs: } 1(5) - (-2)(7) = 5 + 14 = 19 \]
\[ \checkmark: 1(5) - (-2)(13) = 5 + 26 = 31 \]

How to perform the actual pivot

\[
\begin{bmatrix}
    1 & 2 & 0 & | & 4 \\
    0 & 1 & 0 & | & 3 \\
    0 & 0 & 1 & | & 2
\end{bmatrix}
\]

The TI-83 has two functions ref() and rref() that place a matrix into row-echelon form and reduced row-echelon form. Derive provides the row_reduce command. Unfortunately, neither one of these utilities will show work.

Technology?

The instructor has written a pivot program for the TI-82, 83, and 85 calculators that will show intermediate matrices.