

# Mathematical Notation

## Math 121 - Calculus & Analytic Geometry I

Use Word or WordPerfect to recreate the following documents. Each article is worth 10 points and should be emailed to the instructor at james@richland.edu. If you use Microsoft Works to create the documents, then you must print it out and give it to the instructor as he can't open those files.

Type your name at the top of each document.

Include the title as part of what you type. The lines around the title aren't that important, but if you will type ----- at the beginning of a line and hit enter, both Word and WordPerfect will draw a line across the page for you.

For expressions or equations, you should use the equation editor in Word or WordPerfect. The instructor used WordPerfect and a 14 pt Times New Roman font with 0.75" margins, so they may not look exactly the same as your document. The equations were created using 14 pt font.

For individual symbols ( $\mu$ ,  $\sigma$ , etc) within the text of a sentence, you can insert symbols. In Word, use "Insert / Symbol" and choose the Symbol font. For WordPerfect, use Ctrl-W and choose the Greek set. However, it's often easier to just use the equation editor as expressions are usually more complex than just a single symbol. If there is an equation, put both sides of the equation into the same equation editor box instead of creating two objects.

There are instructions on how to use the equation editor in a separate document or on the website. Be sure to read through the help it provides. There are some examples at the end that walk students through the more difficult problems. You will want to read the handout on using the equation editor if you have not used this software before.

If you fail to type your name on the page, you will lose 1 point. Don't type the hints or the reminders at the bottom of each page.

These notations are due at the beginning of class on the day of the exam for that chapter. That is, the chapter 1 notation is due on the day of the chapter 1 test. Late work will be accepted but will lose 20% of its value per class period.

## Chapter 1 - Trigonometry Review

Hint: Create a matrix with 5 rows and 6 columns to make this table

Degrees	0°	30°	45°	60°	90°
Radians	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$\sin \theta$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
$\cos \theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
$\tan \theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	undef

An angle  $\theta$  in QI becomes  $\pi - \theta$  in QII,  $\pi + \theta$  in QIII, and  $2\pi - \theta$  in QIV.

$$\sin(-x) = -\sin x \quad \cos(-x) = \cos x \quad \tan(-x) = -\tan x$$

$$\sin^2 x + \cos^2 x = 1 \quad 1 + \tan^2 x = \sec^2 x \quad \cot^2 x + 1 = \csc^2 x$$

$$\sin\left(\frac{\pi}{2} - x\right) = \cos x \quad \cos\left(\frac{\pi}{2} - x\right) = \sin x \quad \tan\left(\frac{\pi}{2} - x\right) = \cot x$$

$$\cos(a \pm b) = \cos a \cos b \mp \sin a \sin b$$

$$\sin(a \pm b) = \sin a \cos b \pm \cos a \sin b$$

$$\tan(a \pm b) = \frac{\tan a \pm \tan b}{1 \mp \tan a \tan b}$$

$$\cos 2x = \cos^2 x - \sin^2 x \quad \sin 2x = 2 \sin x \cos x \quad \tan 2x = \frac{2 \tan x}{1 - \tan^2 x}$$

$$\cos^2 x = \frac{1 + \cos 2x}{2} \quad \sin^2 x = \frac{1 - \cos 2x}{2}$$

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## Chapter 2 - Limits

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When finding a finite limit, simply substitute the value into the expression unless it causes problems.

The two sided limit  $\lim_{x \rightarrow a} f(x)$  exists if and only if both one sided limits  $\lim_{x \rightarrow a^+} f(x)$  and  $\lim_{x \rightarrow a^-} f(x)$  exist and are equal to each other.

If a rational function has a limit of the form  $0/0$ , then there is a common factor in both the numerator and the denominator. Factor both, reduce, and then evaluate the limit.

When finding infinite limits of polynomial and rational functions, only the leading term needs to be considered. This is only true for limits as  $x \rightarrow +\infty$  or  $x \rightarrow -\infty$ .

$$\lim_{x \rightarrow \infty} (a_n x^n + a_{n-1} x^{n-1} + \cdots + a_0) = \lim_{x \rightarrow \infty} a_n x^n$$
$$\lim_{x \rightarrow \infty} \left( \frac{a_n x^n + a_{n-1} x^{n-1} + \cdots + a_0}{b_m x^m + b_{m-1} x^{m-1} + \cdots + b_0} \right) = \lim_{x \rightarrow \infty} \frac{a_n x^n}{b_m x^m}$$

Definition of a Limit

$\lim_{x \rightarrow a} f(x) = L$  if  $\forall \varepsilon > 0, \exists \delta > 0 \ni |f(x) - L| < \varepsilon$  whenever  $0 < |x - a| < \delta$ .

Common Trigonometric Limits

$$\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1 \quad \lim_{x \rightarrow 0} \frac{1 - \cos x}{x} = 0 \quad \lim_{x \rightarrow 0} \frac{\tan x}{x} = 1$$

A function  $f$  is continuous at  $x = a$  if 1)  $f(a)$  is defined, 2)  $\lim_{x \rightarrow a} f(x)$  exists, and 3)

$$\lim_{x \rightarrow a} f(x) = f(a).$$

If  $f$  is continuous on  $[a, b]$  and  $k$  is between  $f(a)$  and  $f(b)$ , then there exists at least one  $x \in [a, b]$  such that  $f(x) = k$ .

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## Chapter 3 - Derivatives

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Notation

$$\frac{d}{dx}[f(x)] = f'(x) = D_x[f(x)] = \frac{dy}{dx} = y'$$
$$\frac{d^2}{dx^2}[f(x)] = f''(x) = D_{xx}[f(x)] = \frac{d^2y}{dx^2} = y''$$

Definition

$$f'(x) = \frac{d}{dx}[f(x)] = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

Power Rule

$$\frac{d}{dx}[x^n] = n \cdot x^{n-1}$$

Product Rule

$$[f \cdot g]' = f \cdot g' + f' \cdot g$$

Quotient Rule

$$\left(\frac{f}{g}\right)' = \frac{g \cdot f' - f \cdot g'}{g^2}$$

Chain Rule

$$[f(g(x))]' = f'(g(x)) \cdot g'(x)$$
$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

Trigonometric Functions

$$\frac{d}{dx}[\sin x] = \cos x \quad \frac{d}{dx}[\tan x] = \sec^2 x \quad \frac{d}{dx}[\sec x] = \sec x \tan x$$
$$\frac{d}{dx}[\cos x] = -\sin x \quad \frac{d}{dx}[\cot x] = -\csc^2 x \quad \frac{d}{dx}[\csc x] = -\csc x \cot x$$

Local Linear Approximation

$$f(x) \approx f(x_0) + f'(x_0) \cdot \Delta x$$

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## Chapter 4 - Applications of the Derivative

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If  $f$  is differentiable, then  $f$  is increasing when  $f'(x) > 0$ , decreasing when  $f'(x) < 0$ , and constant when  $f'(x) = 0$ .

Critical points occur where  $f'(x) = 0$  or  $f'(x)$  is undefined. Stationary points are the critical points where  $f'(x) = 0$ .

If  $f$  is twice differentiable, then  $f$  is concave up when  $f''(x) > 0$  and concave down when  $f''(x) < 0$ .

Inflection points occur when concavity changes. This can occur when  $f''(x) = 0$  or  $f''(x)$  is undefined.

Relative extrema can only occur at critical points.

If  $f$  is twice differentiable at  $x = a$  and  $f'(a) = 0$ , then there will be a relative minimum at  $x = a$  if  $f''(a) > 0$  and a relative maximum at  $x = a$  if  $f''(a) < 0$ . If  $f''(a) = 0$ , the second derivative test is inconclusive.

### Rectilinear Motion

Position  $s(t)$

Velocity  $v(t) = s'(t) = \frac{ds}{dt}$

Speed  $speed = |v(t)| = \left| \frac{ds}{dt} \right|$

Acceleration  $a(t) = v'(t) = \frac{dv}{dt} = s''(t) = \frac{d^2s}{dt^2}$

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## Chapter 5 - Integration

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$$\frac{d}{dx} \left[ \int f(x) dx \right] = f(x)$$

$$\int x^n dx = \frac{1}{n+1} x^{n+1} + C$$

$$\sum_{k=1}^n (a_k \pm b_k) = \sum_{k=1}^n a_k \pm \sum_{k=1}^n b_k$$

$$\int_a^b f(x) dx = \lim_{\max \Delta x_k \rightarrow 0} \sum_{k=1}^n f(x_k^*) \Delta x_k$$

$$\int_a^a f(x) dx = 0$$

$$\int_b^a f(x) dx = - \int_a^b f(x) dx$$

$$\int_a^b f(x) dx = \int_a^c f(x) dx + \int_c^b f(x) dx$$

If  $f$  is continuous on  $[a, b]$  and  $F$  is any antiderivative of  $f$  on  $[a, b]$ , then

$$\int_a^b f(x) dx = F(x) \Big|_a^b = F(b) - F(a)$$

If  $f$  is continuous and  $F(x) = \int_a^x f(t) dt$  is an antiderivative of  $f$ , then

$$\frac{d}{dx} \left[ \int_a^x f(t) dt \right] = f(x)$$

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## Chapter 6 - Applications of Integration

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Area between two curves

$$A = \int_a^b [f(x) - g(x)] dx$$

Volume of solid of revolution about  $x$ -axis by disk method

$$V = \int_a^b \pi [f(x)]^2 dx$$

Volume of solid of revolution by washer method ( $x$ -axis)

$$V = \int_a^b \pi ([f(x)]^2 - [g(x)]^2) dx$$

Volume of solid of revolution by cylindrical shell method ( $y$ -axis)

$$V = \int_a^b 2\pi \cdot x \cdot f(x) dx$$

Length of plane curve

$$L = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt$$

Area of a surface of revolution ( $x$ -axis)

$$S = \int_a^b 2\pi \cdot f(x) \sqrt{1 + [f'(x)]^2} dx$$

Average Value

$$f_{ave} = \frac{1}{b-a} \int_a^b f(x) dx$$

Work

$$W = \int_a^b F(x) dx$$

Fluid Force

$$F = \int_a^b \rho \cdot h(x) \cdot w(x) dx$$