Mathematical Notation

Math 160 - Finite Mathematics

Use Word or WordPerfect to recreate the following documents. Each article is worth 10 points and should be emailed to the instructor at james@richland.edu.

Type your name at the top of each document.

Include the title as part of what you type. The lines around the title aren't that important, but if you will type ----- at the beginning of a line and hit enter, both Word and WordPerfect will draw a line across the page for you.

For expressions or equations, you should use the equation editor in Word or WordPerfect. The instructor used WordPerfect and a 14 pt Times New Roman font with standard 0.75" margins, so they may not look exactly the same as your document. The equations were created using 16 pt font, but feel free to use a smaller font.

If there is an equation, put both sides of the equation into the same equation editor box instead of creating two objects. Be sure to use the proper symbols, there are some instances where more than one symbol may look the same, but they have different meanings and don't appear the same as what's on the assignment.

There are instructions on how to use the equation editor in a separate document or on the website. Be sure to read through the help it provides. There are some examples at the end that walk students through the more difficult problems. You will want to read the handout on using the equation editor if you have not used this software before.

If you fail to type your name on the document, you will lose 1 point. Don't type the hints or reminders that appear on the pages.

These notations are due before the beginning of class on the day of the exam for that chapter. That is, the chapter 3 notation is due on the day of the chapter 3 test. Late work will be accepted but will lose 20% of its value per class period. If I receive your emailed assignment more than one class period before it is due and you don't receive all 10 points, then I will email you back with things to correct so that you can get all the points. Any corrections need to be submitted by the due date and time or the original score will be used.

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Chapter 3 - Finance

Simple Interest

One payment, interest is not compounded

$$I = PRT$$

I = Interest, P = Principal, R = Rate, T = Time

Compound Interest

One payment, interest is compounded

$$A = P\left(1+i\right)^{n}$$

A = Amount, P = Principal, i = Periodic rate, n = Number of periods

Future Value Annuities

A series of payments where the balance grows in value over time

$$FV = PMT\left(\frac{\left(1+i\right)^{n}-1}{i}\right)$$

FV = Future value, PMT = Payment, i = Periodic rate, n = Number of periods

Present Value Annuities

A series of payments where the balance decreases in value over time

$$PMT = PV\left(\frac{i}{1-\left(1+i\right)^{-n}}\right)$$

PV = Present value, PMT = Payment, i = Periodic rate, n = Number of periods

Chapter 4 - Systems of Equations, Matrices

Operations that produce row equivalent matrices

1. Switch two rows of a matrix
2. Multiply a row by a non-zero constant
3. Add a constant multiple of one row to another row

Augmented matrix in reduced row-echelon form

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & -3 \end{array}\right] \Rightarrow \begin{array}{l} x_1 = 2 \\ x_2 = 4 \\ x_3 = -3 \end{array}$$

Matrix Multiplication

$$\begin{bmatrix} 3 & -1 & 2 \\ 4 & 7 & -5 \end{bmatrix} \begin{bmatrix} 6 & 1 \\ 2 & -5 \\ -3 & 2 \end{bmatrix} = \begin{bmatrix} 10 & 12 \\ 53 & -41 \end{bmatrix}$$

Solving a system of linear equations using matrix inverses

$$\mathbf{AX} = \mathbf{B} \Rightarrow \mathbf{X} = \mathbf{A}^{-1}\mathbf{B}$$

Leontief Input-Output Model

$$\mathbf{X} = (\mathbf{I} - \mathbf{M})^{-1}\mathbf{D}$$

X = Output matrix
M = Technology Matrix
D = Demand Matrix

Chapter 5 - Linear Inequalities

Hint, place following two systems into a matrix without brackets. Choose Format / Define Spacing and set the matrix row spacing to be 120% and the matrix column spacing to be 50%.

System of linear inequalities

$$\begin{matrix} 3x & + & 4y & \le & 36 \\ 3x & + & 2y & \le & 30 \\ & & x & \ge & 0 \\ & & y & \ge & 0 \end{matrix}$$

Existence of Solutions to a Linear Programming Problem

- For a bounded feasible region, the objective function will always have both a maximum and minimum value of the objective function.
- For an unbounded feasible region with positive coefficients of the objective function, there will be a maximum but no minimum value.
- If the feasible region is empty, then the objective function has no maximum or minimum value.

Fundamental Theorem of Linear Programming

If there is a solution to a linear programming problem, then it will occur at one or more corner points of the feasible region or on the boundary between two corner points.

Although usually not written in the problem itself, almost every story problem has non-negativity constraints. These state that the variables cannot be negative and are written as $x \ge 0$ and $y \ge 0$. These non-negativity constraints limit us to the first quadrant.

Don't forget to put your name at the top of the page

Chapter 6 - Linear Programming

A standard maximization problem requires all problem constraints to be in the form of ≤ a non-negative constant, but the objective function coefficients can be any real number.

A standard minimization problem requires all problem constraints to be in the form of ≥ any real number, but the objective function coefficients cannot be negative.

Hint, place following two systems into a matrix without brackets. Choose Format / Define Spacing and set the matrix row spacing to be 120% and the matrix column spacing to be 50%.

Standard maximization **problem**

Maximize $P = 100x_1 + 300x_2 + 200x_3$

subject to

$$\begin{matrix} x_1 & + & x_2 & + & x_3 & \leq & 100 \\ 40x_1 & + & 20x_2 & + & 30x_3 & \leq & 3200 \\ x_1 & + & 2x_2 & + & x_3 & \leq & 160 \\ x_1 & , & x_2 & , & x_3 & \geq & 0 \end{matrix}$$

Initial system for a standard maximization problem after adding slack variables.

Maximize $P = 100x_1 + 300x_2 + 200x_3$

subject to

$$\begin{matrix} x_1 & + & x_2 & + & x_3 & + & s_1 & & & & & = & 100 \\ 40x_1 & + & 20x_2 & + & 30x_3 & & & + & s_2 & & & = & 3200 \\ x_1 & + & 2x_2 & + & x_3 & & & & & + & s_3 & = & 160 \end{matrix}$$

Be sure to reset the matrix row spacing to 150% and the matrix column spacing to 100%.

Initial tableau after moving the objective function to the left side of the equation.

$$\left[\begin{array}{ccccccc|c} 1 & 1 & 1 & 1 & 0 & 0 & 0 & 100 \\ 40 & 20 & 30 & 0 & 1 & 0 & 0 & 3200 \\ 1 & 2 & 1 & 0 & 0 & 1 & 0 & 160 \\ \hline -100 & -300 & -200 & 0 & 0 & 0 & 1 & 0 \end{array}\right]$$

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Chapter 7 - Logic, Sets, and Counting

Truth Tables

p	q	and $p \wedge q$	or $p \vee q$	conditional $p \rightarrow q$	converse $q \rightarrow p$	contrapositive $\neg q \rightarrow \neg p$
T	T	T	T	T	T	T
T	F	F	T	F	T	F
F	T	F	T	T	F	T
F	F	F	F	T	T	T

If **A** = { 1, 2, 4, 6 } and **B** = { 2, 3, 5 }, then ...

... the union of the sets is $\mathbf{A} \cup \mathbf{B} = \{1, 2, 3, 4, 5, 6\}$

... the intersection of the sets is $\mathbf{A} \cap \mathbf{B} = \{2\}$

Fundamental Counting Principle

The total number of ways that two events can happen is found by multiplying together the number of ways that each event can happen.

Permutations

A permutation is an arrangement of objects without repetition but with regard to order.

$${}_nP_r = \frac{n!}{(n-r)!}$$

Combinations

A combination is an arrangement of objects without repetition and without regard to order.

$${}_nC_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}$$

Chapter 8 - Probability

Probability formulas

Addition Rule

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Multiplication Rule

$$P(A \cap B) = P(A)P(B \mid A)$$

Conditional Probability

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

Complement of an Event

$$P(E') = 1 - P(E)$$

Expected value (mean or average)

The expected value is found by multiplying values by their probabilities and then adding.

$$E(x) = \sum_{k=1}^{n} x_k p(x_k)$$

Decision Theory

Expected value (Bayesian) criterion. Find the expected value under each action and choose the action with the largest expected value.

Maximax criterion. Find the maximum payoff under each action and then choose the action with the largest best case scenario.

Maximin criterion. Find the minimum payoff under each action and then choose the action with the largest worst case scenario.

Minimax criterion. Find the opportunistic loss for each state of nature. Then find the maximum opportunistic loss for each action and choose the action with the smallest maximum loss.

Chapter 9 - Markov Chains

S is a state matrix, **P** is the transition matrix. Both are probability matrices, meaning the sum of each row is 1.

Regular Markov chains

$\mathbf{S}_0$ is the initial state matrix. It reflects the beginning conditions.

$$\mathbf{S}_1 = \mathbf{S}_0\mathbf{P}$$

$$\mathbf{S}_2 = \mathbf{S}_1\mathbf{P} = \mathbf{S}_0\mathbf{P}^2$$

$$\mathbf{S}_3 = \mathbf{S}_2\mathbf{P} = \mathbf{S}_0\mathbf{P}^3$$

Steady State Matrix

$$\mathbf{S} = \mathbf{SP}$$

If the Markov chain is regular, then there is a limiting matrix $\overline{\mathbf{P}} = \mathbf{P}^{\infty}$ where each row is the Steady State matrix.

Absorbing Markov Chains

$$\mathbf{P} = \left[\begin{array}{c|c} \mathbf{I} & \mathbf{0} \\ \hline \mathbf{R} & \mathbf{Q} \end{array}\right]$$

Fundamental Matrix **F** (expected frequencies)

$$\mathbf{F} = \left(\mathbf{I} - \mathbf{Q}\right)^{-1}$$

The element in row R, column C of the fundamental matrix represents the expected number of times you will spend in state transient C of the system before ending up at some absorbing state if you start in transient state R.

Limiting Matrix (long term probabilities)

$$\overline{\mathbf{P}} = \mathbf{P}^{\infty} = \left[\begin{array}{c|c} \mathbf{I} & \mathbf{0} \\ \hline \mathbf{FR} & \mathbf{0} \end{array}\right]$$

The element in row R, column C of the matrix **FR** represents the long term probability of ending up in absorbing state C if you started in transient state R.

Chapter 10 - Game Theory

If the game matrix is $\mathbf{M} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ and $D = (a+d) - (b+c)$, then the solution to a two player, zero-sum, non-strictly determined game is

$$\mathbf{P}^* = \begin{bmatrix} \frac{d-c}{D} & \frac{a-b}{D} \end{bmatrix} \quad \mathbf{Q}^* = \begin{bmatrix} \frac{d-b}{D} \\ \frac{a-c}{D} \end{bmatrix} \quad v = \frac{ad-bc}{D}$$

Linear Programming Problem

Assume that $\mathbf{M} = \begin{bmatrix} a & b \\ c & d \\ e & f \end{bmatrix}$ has all positive entries (add a constant if it doesn't).

The optimal row player strategy $\mathbf{P}^* = \begin{bmatrix} p_1 & p_2 & p_3 \end{bmatrix}$ is found by solving:

Minimize $z = \frac{1}{v} = x_1 + x_2 + x_3$ where $x_1 = \frac{p_1}{v}$, $x_2 = \frac{p_2}{v}$, and $x_3 = \frac{p_3}{v}$

Subject to

$$\begin{array}{ccccccc} ax_1 & + & cx_2 & + & ex_3 & \geq & 1 \\ bx_1 & + & dx_2 & + & fx_3 & \geq & 1 \\ x_1 & , & x_2 & , & x_3 & \geq & 0 \end{array}$$

The optimal column player solution $\mathbf{Q}^* = \begin{bmatrix} q_1 \\ q_2 \end{bmatrix}$ is found by solving:

Maximize $z = \frac{1}{v} = y_1 + y_2$ where $y_1 = \frac{q_1}{v}$ and $y_2 = \frac{q_2}{v}$

Subject to

$$\begin{array}{ccccc} ay_1 & + & by_2 & \leq & 1 \\ cy_1 & + & dy_2 & \leq & 1 \\ ey_1 & + & fy_2 & \leq & 1 \\ y_1 & , & y_2 & \geq & 0 \end{array}$$