

# Math 121: Mathematical Notation

## Purpose:

One goal in any course is to properly use the language of that subject. Calculus is no different and may often seem like a foreign language. These notations summarize some of the major concepts and more difficult topics of the unit. Typing them helps you learn the material while teaching you to properly express mathematics on the computer. Part of your grade is for *properly* using mathematical content.

## Instructions:

Use Word or WordPerfect to recreate the following documents. Each article is worth 10 points and should be emailed to the instructor at [james@richland.edu](mailto:james@richland.edu). This is not a group assignment, each person needs to create and submit their own notation.

Type your name at the top of each document. Include the title as part of what you type. The lines around the title aren't that important, but if you will type ----- at the beginning of a line and hit enter, both Word and WordPerfect will convert it to a line.

For expressions or equations, you should use the equation editor in Word or WordPerfect. The instructor used WordPerfect and a 14 pt Times New Roman font with 0.75" margins, so they may not look exactly the same as your document.

If there is an equation, put both sides of the equation into the same equation editor box instead of creating two objects. Be sure to use the proper symbols, there are some instances where more than one symbol may look the same, but they have different meanings and don't appear the same as what's on the assignment. There are some useful tips on the website at <http://people.richland.edu/james/editor/>

**If you fail to type your name on the document, you will lose 1 point.** Don't type **the hints or reminders** that appear on the pages.

These notations are due before the beginning of class on the day of the exam for that material. For example, notation 1 is due on the day of exam 1. Late work will be accepted but will lose 20% of its value per class period. If I receive your emailed assignment more than one class period before it is due and you don't receive all 10 points, then I will email you back with things to correct so that you can get all the points. Any corrections need to be submitted by the due date and time or the original score will be used.

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## Chapter 1 - Trigonometry Review

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Degrees	$0^\circ$	$30^\circ$	$45^\circ$	$60^\circ$	$90^\circ$
Radians	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$
$\sin \theta$	$\sqrt{0}/2 = 0$	$\sqrt{1}/2 = 1/2$	$\sqrt{2}/2$	$\sqrt{3}/2$	$\sqrt{4}/2 = 1$
$\cos \theta$	$\sqrt{4}/2 = 1$	$\sqrt{3}/2$	$\sqrt{2}/2$	$\sqrt{1}/2 = 1/2$	$\sqrt{0}/2 = 0$
$\tan \theta$	0	$1/\sqrt{3}$	1	$\sqrt{3}$	undefined

An angle  $\theta$  in QI becomes  $\pi - \theta$  in QII,  $\pi + \theta$  in QIII, and  $2\pi - \theta$  or  $-\theta$  in QIV.

$$\sin(-x) = -\sin x \quad \cos(-x) = \cos x \quad \tan(-x) = -\tan x$$

$$\sin^2 x + \cos^2 x = 1 \quad 1 + \tan^2 x = \sec^2 x \quad \cot^2 x + 1 = \csc^2 x$$

$$\sin\left(\frac{\pi}{2} - x\right) = \cos x \quad \cos\left(\frac{\pi}{2} - x\right) = \sin x \quad \tan\left(\frac{\pi}{2} - x\right) = \cot x$$

$$\cos(a \pm b) = \cos a \cos b \mp \sin a \sin b$$

$$\sin(a \pm b) = \sin a \cos b \pm \cos a \sin b$$

$$\tan(a \pm b) = \frac{\tan a \pm \tan b}{1 \mp \tan a \tan b}$$

$$\cos 2x = \cos^2 x - \sin^2 x \quad \sin 2x = 2 \sin x \cos x \quad \tan 2x = \frac{2 \tan x}{1 - \tan^2 x}$$

$$\cos^2 x = \frac{1 + \cos 2x}{2} \quad \sin^2 x = \frac{1 - \cos 2x}{2}$$

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## Chapter 2 - Limits

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When finding a finite limit, simply substitute the value into the expression unless it causes problems.

The two sided limit  $\lim_{x \rightarrow a} f(x)$  exists if and only if both one sided limits  $\lim_{x \rightarrow a^+} f(x)$  and  $\lim_{x \rightarrow a^-} f(x)$  exist and are equal to each other.

If a rational function has a limit of the form  $0/0$ , then there is a common factor in both the numerator and the denominator. Factor both, reduce, and then evaluate the limit.

When finding infinite limits of polynomial and rational functions, only the leading term needs to be considered. This is only true for limits as  $x \rightarrow +\infty$  or  $x \rightarrow -\infty$ .

$$\lim_{x \rightarrow \infty} (a_n x^n + a_{n-1} x^{n-1} + \dots + a_0) = \lim_{x \rightarrow \infty} a_n x^n$$
$$\lim_{x \rightarrow \infty} \left( \frac{a_n x^n + a_{n-1} x^{n-1} + \dots + a_0}{b_m x^m + b_{m-1} x^{m-1} + \dots + b_0} \right) = \lim_{x \rightarrow \infty} \frac{a_n x^n}{b_m x^m}$$

Definition of a Limit

$\lim_{x \rightarrow a} f(x) = L$  if  $\forall \varepsilon > 0, \exists \delta > 0 \ni |f(x) - L| < \varepsilon$  whenever  $0 < |x - a| < \delta$ .

Common Trigonometric Limits

$$\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1 \quad \lim_{x \rightarrow 0} \frac{1 - \cos x}{x} = 0 \quad \lim_{x \rightarrow 0} \frac{\tan x}{x} = 1$$

A function  $f$  is continuous at  $x = a$  if 1)  $f(a)$  is defined, 2)  $\lim_{x \rightarrow a} f(x)$  exists, and 3)

$$\lim_{x \rightarrow a} f(x) = f(a).$$

If  $f$  is continuous on  $[a, b]$  and  $k$  is between  $f(a)$  and  $f(b)$ , then there exists at least one  $x \in [a, b]$  such that  $f(x) = k$ .

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## Chapter 3 - Derivatives

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These don't have to be aligned like this, I just did it so it would fit on one page.

Notation  $\frac{d}{dx}[f(x)] = f'(x) = D_x[f(x)] = \frac{dy}{dx} = y'$

$$\frac{d^2}{dx^2}[f(x)] = f''(x) = D_{xx}[f(x)] = \frac{d^2y}{dx^2} = y''$$

Definition  $f'(x) = \frac{d}{dx}[f(x)] = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$

Power Rule  $\frac{d}{dx}[x^n] = n \cdot x^{n-1}$

Product Rule  $[f \cdot g]' = f \cdot g' + f' \cdot g$

Quotient Rule  $\left(\frac{f}{g}\right)' = \frac{g \cdot f' - f \cdot g'}{g^2}$

Chain Rule  $[f(g(x))]' = f'(g(x)) \cdot g'(x)$   
 $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$

Trigonometric Functions

$$\frac{d}{dx}[\sin x] = \cos x \quad \frac{d}{dx}[\tan x] = \sec^2 x \quad \frac{d}{dx}[\sec x] = \sec x \tan x$$

$$\frac{d}{dx}[\cos x] = -\sin x \quad \frac{d}{dx}[\cot x] = -\csc^2 x \quad \frac{d}{dx}[\csc x] = -\csc x \cot x$$

Local Linear Approximation  $f(x) \approx f(x_0) + f'(x_0) \cdot \Delta x$

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## Chapter 4 - Applications of the Derivative

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If  $f$  is differentiable, then  $f$  is increasing when  $f'(x) > 0$ , decreasing when  $f'(x) < 0$ , and constant when  $f'(x) = 0$ .

Critical points occur where  $f'(x) = 0$  or  $f'(x)$  is undefined. Stationary points are the critical points where  $f'(x) = 0$ .

If  $f$  is twice differentiable, then  $f$  is concave up when  $f''(x) > 0$  and concave down when  $f''(x) < 0$ .

Inflection points occur when concavity changes. This can occur when  $f''(x) = 0$  or  $f''(x)$  is undefined.

Relative extrema can only occur at critical points.

If  $f$  is twice differentiable at  $x = a$  and  $f'(a) = 0$ , then there will be a relative minimum at  $x = a$  if  $f''(a) > 0$  and a relative maximum at  $x = a$  if  $f''(a) < 0$ . If  $f''(a) = 0$ , the second derivative test is inconclusive.

### Rectilinear Motion

Position  $s(t)$

Velocity  $v(t) = s'(t) = \frac{ds}{dt}$

Speed  $speed = |v(t)| = \left| \frac{ds}{dt} \right|$

Acceleration  $a(t) = v'(t) = \frac{dv}{dt} = s''(t) = \frac{d^2s}{dt^2}$

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## Chapter 5 - Integration

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$$\frac{d}{dx} \left[ \int f(x) dx \right] = f(x)$$

$$\int x^n dx = \frac{1}{n+1} x^{n+1} + C$$

$$\sum_{k=1}^n (a_k \pm b_k) = \sum_{k=1}^n a_k \pm \sum_{k=1}^n b_k$$

$$\int_a^b f(x) dx = \lim_{\max \Delta x_k \rightarrow 0} \sum_{k=1}^n f(x_k^*) \Delta x_k$$

$$\int_a^a f(x) dx = 0$$

$$\int_b^a f(x) dx = -\int_a^b f(x) dx$$

$$\int_a^b f(x) dx = \int_a^c f(x) dx + \int_c^b f(x) dx$$

If  $f$  is continuous on  $[a, b]$  and  $F$  is any antiderivative of  $f$  on  $[a, b]$ , then

$$\int_a^b f(x) dx = F(x) \Big|_a^b = F(b) - F(a)$$

If  $f$  is continuous and  $F(x) = \int_a^x f(t) dt$  is an antiderivative of  $f$ , then

$$\frac{d}{dx} \left[ \int_a^x f(t) dt \right] = f(x)$$

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## Chapter 6 - Applications of Integration

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To get the  $\int$  symbol to grow with the integrand, hold down the shift key when selecting it from the menu.

Area between two curves

$$A = \int_a^b [f(x) - g(x)] dx = \int_{\text{left}}^{\text{right}} (\text{top} - \text{bottom}) dx$$

Volume of solid of revolution about  $x$ -axis using disk method

$$V = \int_a^b \pi [f(x)]^2 dx$$

Volume of solid of revolution about  $x$ -axis using washer method

$$V = \int_a^b \pi ([f(x)]^2 - [g(x)]^2) dx$$

Volume of solid of revolution about  $y$ -axis using cylindrical shell method

$$V = \int_a^b 2\pi \cdot x \cdot f(x) dx$$

Length of plane curve

$$L = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt$$

Area of a surface of revolution ( $x$ -axis)

$$S = \int_a^b 2\pi \cdot f(x) \sqrt{\left(\frac{dx}{dx}\right)^2 + \left(\frac{dy}{dx}\right)^2} dx$$

Average Value

$$f_{\text{ave}} = \frac{1}{b-a} \int_a^b f(x) dx$$

Work

$$W = \int_a^b F(x) dx$$

Fluid Force

$$F = \int_a^b \rho \cdot h(x) \cdot w(x) dx$$