Math 122 – Chapter 8 Take Home Name : _____
Spring 2011 50 points

Use extra paper for work and attach with test. Write the answers on this sheet. Each question is worth 2 points.

1. The Laplace transform, $\mathcal{L}\{f(x)\}$, is useful in differential equations. Find the following Laplace transforms. Note that the Laplace Transform is a function of *s* only, there is no *x* in the answer. The restrictions on *s* that are given are necessary so that the improper integral converges, be sure to state proper restrictions. Most of the integrals will require integration by parts if done by hand, however you may use the table of integrals in Appendix B. An example problem has been worked for you.

$$\mathcal{L}\{f(x)\} = \int_0^{+\infty} e^{-sx} f(x)\,dx$$

Evaluate any five (5) of the following transforms.

a. $\mathcal{L}\{a\}$

b. $\mathcal{L}\{ax\}$

c. $\mathcal{L}\{\sin ax\}$

d. $\mathcal{L}\{\cos ax\}$

e. $\mathcal{L}\{\sinh ax\}$

f. $\mathcal{L}\{\cosh ax\}$

g. $\mathcal{L}\{xe^{-ax}\}$

Example: Find $\mathcal{L}\{e^{ax}\}$

$$\mathcal{L}\{e^{ax}\} = \int_0^{+\infty} e^{-sx} e^{ax}\,dx$$

$$= \int_0^{+\infty} e^{-(s-a)x}\,dx$$

let $u = -(s-a)x$, $du = -(s-a)dx$

$u\big|_{x=0} = 0$

if $s > a$, then $u\big|_{x\to+\infty} \to -\infty$

$$= \frac{-1}{s-a}\int_0^{-\infty} e^u\,du = \frac{1}{s-a}\int_{-\infty}^{0} e^u\,du$$

$$= \frac{e^u}{s-a}\Big|_{-\infty}^{0} = \frac{1}{s-a}\left(e^0 - \lim_{b\to-\infty} e^b\right)$$

$$= \frac{1-0}{s-a}$$

$$\mathcal{L}\{e^{ax}\} = \frac{1}{s-a},\ s > a$$

2. Use the table of integrals in Appendix B to find the integral. In each case, copy the number of the formula and the integration formula itself. Then give the values of any variables (ex: *a* or *u*). Finally, find the integral and simplify.

   Here is an example solution for $\int \frac{1}{x\sqrt{3x-4}}\,dx$

   The matching formula is #17 $\int \frac{du}{u\sqrt{a+bu}} = \frac{2}{\sqrt{-a}}\tan^{-1}\sqrt{\frac{a+bu}{-a}} + C$

   with $a = -4, u = 3x, du = 3dx$.

   Adjust constants in the original integral to match the formula.

   $\int \frac{3}{3x\sqrt{3x-4}}\,dx$

   Substitute $\frac{2}{\sqrt{-(-4)}}\tan^{-1}\sqrt{\frac{3x-4}{-(-4)}} + C$, and simplify $\tan^{-1}\left(\frac{\sqrt{3x-4}}{2}\right) + C$

   a. Problem 8.6.22

   b. Problem 8.6.24

   c. Problem 8.6.34

   d. Problem 8.6.44

   e. Problem 8.6.66

3. Use a computer algebra system (Maxima, Derive, Maple, Mathematica, or TI-89/92) to find the following integrals. You might factor or expand to simplify.

   a. Problem 8.6.26

   b. Problem 8.6.40

   c. Problem 8.6.50

   d. Problem 8.6.62

   e. Problem 8.6.70

4. Work ten (10) of the following problems by hand. There must be exactly two problems from each section (8.2, 8.3, 8.4, 8.5, and 8.8). Show all work. Clearly identify the section number and problem. Attach the problems in section order. You may use the reduction formulas where necessary, but otherwise do not use the table of integrals. You may use a CAS to check your answer, but show work.

   a. Problem 8.2.16

   b. Problem 8.2.34

   c. Problem 8.2.60

   d. Problem 8.3.16

   e. Problem 8.3.54

   f. Problem 8.3.70

   g. Problem 8.4.30

   h. Problem 8.4.46

   i. Problem 8.4.52

   j. Problem 8.5.14

   k. Problem 8.5.24

   l. Problem 8.5.46

   m. Problem 8.8.24

   n. Problem 8.8.34

   o. Problem 8.8.50