Example Technology Exercise 9
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(%i1)
```
ratprint:false$
```

1 Problem 9.7.33

(%i2)
```
X:[0,0.25,0.50,0.75,1.00]$
T0:makelist(sin(X[k]),k,1,length(X))$
taylor(sin(x),x,0,1)$
T1:makelist(subst(x=X[k],%),k,1,length(X))$
taylor(sin(x),x,0,3)$
T2:makelist(subst(x=X[k],%),k,1,length(X))$
taylor(sin(x),x,0,5)$
T3:makelist(subst(x=X[k],%),k,1,length(X))$
```

Put it into a matrix so it looks like a table.
If you don't want it in a table, then change the $ to ;
at the end of the X, T0, T1, T2, and T3 lines above.

The fpprintprec is to set it to round the printed result
to 8 decimals just so it fits on the page. We reset it
after the matrix.

(%i10)
```
fpprintprec:8$
matrix(X,T0,T1,T2,T3);
fpprintprec:0$
```

(%o11)

0	0.25	0.5	0.75	1.0
0	0.247404	0.479426	0.681639	0.841471
0	0.25	0.5	0.75	1.0
0	0.247396	0.479167	0.679688	0.833333
0	0.247404	0.479427	0.681665	0.841667

2 Problem 9.7.54

Find order of Maclaurin series so error < 0.0001

```
(%i13) F:cos(%pi*x^2)$
       x0:0.6$
```

Since the cosine is an alternating series,
the error is always less than the next term in the series

```
(%i15) makelist(subst(x=0,diff(F,x,k))/k!*x0^k,k,0,16)$
       float(%);
```

(%o16) [1.0,0.0,0.0,0.0,-0.63955036519059,0.0,0.0,0.0, 0.068170778269236,0.0,0.0,0.0,-0.0029065764091611,0.0,0.0,0.0, $6.6389357283333904\ 10^{-5}$]

The error on the 16th order Maclaurin series is less than 0.0001
But the previous term before that is x^12, so we need a 12th order
Maclaurin series to guarantee the error is < 0.0001

```
(%i17) taylor(F,x,0,12);
       subst(x=x0,%)$
       estimate:float(%);
       subst(x=x0,F)$
       actual:float(%);
       err:abs(estimate-actual);
```

(%o17)/T/ $1-\frac{\pi^2 x^4}{2}+\frac{\pi^4 x^8}{24}-\frac{\pi^6 x^{12}}{720}+...$

(%o19) 0.42571383666948

(%o21) 0.42577929156507

(%o22) $6.5454895587957473\ 10^{-5}$

3 Problem 9.7.58

Find values of x such that the maximum error on approximating
sin(x) < 0.001

```
(%i23) F:x-x^3/3!$
       MaxError:0.001$
```

Since this series alternates, the error is less than
than the next term in the series.

(%i25) nextTerm:abs(1/5!*x^5)$
X1:find_root(nextTerm=MaxError,x,-1,-0.1);
X2:find_root(nextTerm=MaxError,x,0.1,1);

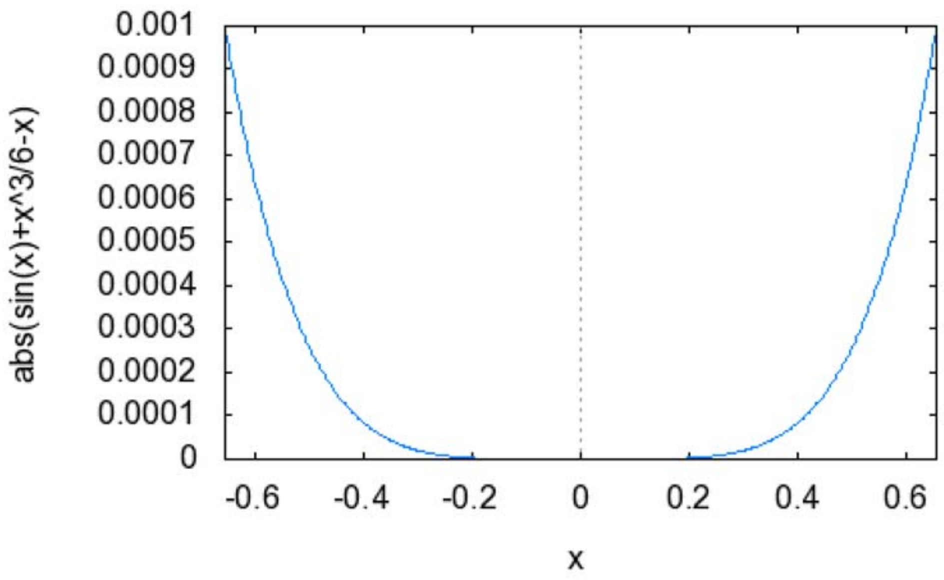
(%o26) -0.65438938994124

(%o27) 0.65438938994124

Graph the error to show it's less than 0.001

(%i28) wxplot2d(abs(F-sin(x)),[x,X1,X2]);

(%t28)



(%o28)

4 Problem 9.10.47

Multiply the individual Maclaurin series and show you get the same thing as the Maclaurin series for the product.

```
(%i29) F:exp(x)$
       G:sin(x)$
       TayF:taylor(F,x,0,4);
       TayG:taylor(G,x,0,5);
       TayF*TayG;
       taylor(F*G,x,0,5);
```

(%o31)/T/ $1+x+\frac{x^2}{2}+\frac{x^3}{6}+\frac{x^4}{24}+...$

(%o32)/T/ $x-\frac{x^3}{6}+\frac{x^5}{120}+...$

(%o33)/T/ $x+x^2+\frac{x^3}{3}-\frac{x^5}{30}+...$

(%o34)/T/ $x+x^2+\frac{x^3}{3}-\frac{x^5}{30}+...$

5 Problem 9.10.51

You could have just divide the Maclaurin series for sin(x) by the polynomial 1+x, but Maxima won't write it as a Taylor series if you do. So it's easier to multiply by the power series for 1/(1+x).

```
(%i35) F:sin(x)$
       G:1/(1+x)$
       TayF:taylor(F,x,0,5);
       TayG:taylor(G,x,0,5);
       TayF*TayG;
       taylor(F*G,x,0,4);
```

(%o37)/T/ $x-\frac{x^3}{6}+\frac{x^5}{120}+...$

(%o38)/T/ $1-x+x^2-x^3+x^4-x^5+...$

(%o39)/T/ $x-x^2+\frac{5x^3}{6}-\frac{5x^4}{6}+\frac{101x^5}{120}+...$

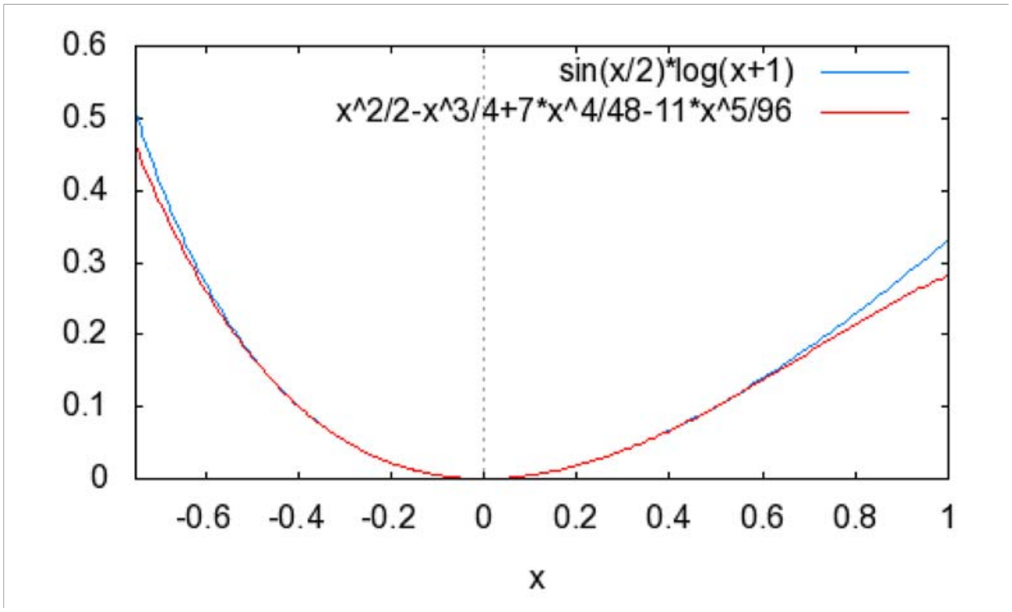
(%o40)/T/ $x-x^2+\frac{5x^3}{6}-\frac{5x^4}{6}+...$

6 Problem 9.10.80

It is necessary to truncate the Taylor series
or Maxima considers F and T the same and returns
an error of 0

```
(%i41) F:sin(x/2)*log(1+x)$
       T:trunc(taylor(F,x,0,5));
       wxplot2d([F,T],[x,-0.75,1]);
```

(%o42) $\frac{x^2}{2}-\frac{x^3}{4}+\frac{7x^4}{48}-\frac{11x^5}{96}+...$

(%t43) 

(%o43)

The approximation is pretty good (less than 0.01)
on approximately (-0.609,0.746)

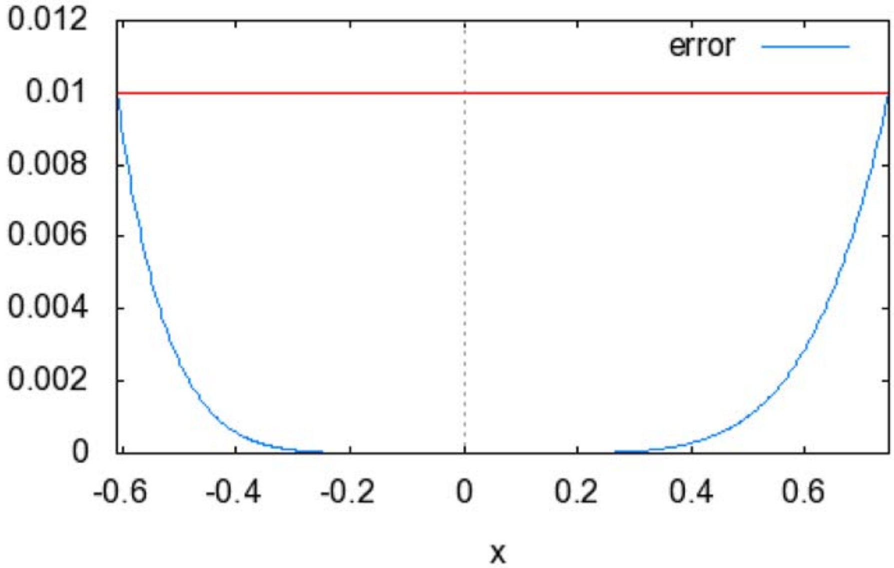
```
(%i44) X1:find_root(abs(F-T)=0.01,x,-0.7,-0.5);
       X2:find_root(abs(F-T)=0.01,x,0,1);
```

(%o44) -0.60920296531085

(%o45) 0.74685165677014

(%i46) wxplot2d([abs(F-T),0.01],[x,X1,X2],[legend,"error",""]);

(%t46)



(%o46)