

# Math 160 - Projects

This course has some projects that require you to go beyond the level of material covered in the book or use outside sources to acquire data.

Although you may work the problems individually, the projects are designed to be group projects. They require extra work and you can benefit from the group. The exams will often see questions similar to those on the projects and so having other people verify the correctness of the problems before you take the exam can be beneficial.

Groups may have up to 3 people in them (except for the first project, which is an individual project). You will be able to select your own groups for the projects and you may change groups at midterm.

Some of these projects are very similar to problems that will appear on your exams. For that reason, it is important that you complete and understand them before taking your exam, even if they are not due until after the exam.

On the following pages, questions that need an answer begin with \_\_\_\_\_. There are templates and entry forms inside Canvas so that you can enter your answers online.

## Project 3A: Planning for Retirement (Chapter 3)

This project is designed to help you get a sense of what is needed to retire. When estimating costs, remember that inflation will occur and things will be more expensive than they are now, but that by the that time , you are likely to have most major purchases like a house or car already purchased.

All ages should be in years, and for simplicity, we will assume that all major events (start saving, retiring, and dying) occur on your birthday so that we don't have to mess with partial years. Also, for purposes of this project, you must not plan on retiring within the next 15 years. Round all dollar amounts to the nearest cent. Assume that all money is invested and earns interest at an annual nominal rate of 7.5%, compounded monthly.

### Information Gathering Worksheet

- \_\_\_\_\_ Age, in years, that you will be on your next birthday
- \_\_\_\_\_ Age, in years, when you plan on retiring and \_\_\_\_\_ years until retirement
- \_\_\_\_\_ Number of years that you expect to live and \_\_\_\_\_ years of retirement
- \_\_\_\_\_ Amount, in dollars, that you anticipate needing *each month* during your retirement

### Planning Document

- \_\_\_\_\_ Amount of money that you need to have saved the day you retired in order to fund your retirement
- \_\_\_\_\_ Amount of money that you will need to save each month until you retire in order for you to reach the amount needed to retire

After 10 years, a trust fund established upon the death of your grandparents gives you a one-time payment of \$125,000. You spend \$75,000 of it to pay off your college and credit card debts and put the rest of it into your retirement fund and continue making your regular deposits until you retire.

- \_\_\_\_\_ Amount of money you will have after 10 years of regular payments into your retirement fund
- \_\_\_\_\_ Amount of money the \$50,000 from the trust fund will grow to before you retire
- \_\_\_\_\_ Amount of money you'll be able to spend each month during retirement when you include the investment results from the trust fund

## Project 3B: Buying a Home (Chapter 3)

The purpose of this project is to help you understand purchasing a home. You will make a 20% down payment to avoid escrow costs and finance the balance over 30 years with a fixed loan rate of 4.2%. Payments and compounding will occur monthly. Round all dollar amounts to the nearest cent.

Find a home in the Richland district that is between \$80,000 and \$200,000 by visiting <http://www.realtor.com> or <https://www.zillow.com> or some other online real-estate system.

----- Address of home

----- Asking price of home

----- Amount financed after 20% down payment

----- Monthly payment needed to finance home

----- Total cost to repay loan if regular payments are made for the full 30 years (the loan does not include the down payment)

----- Amount of interest paid if the loan goes full term

For the next two questions, assume that ten years of regular payments have been made.

----- Amount still owed on the house

----- Equity in home if the house is appraised at 8% more than you originally paid for it

For the next two questions, assume that from the beginning of the loan, you have paid an extra \$250 per month. This section has nothing to do with the 10 years in the previous two questions.

----- The number of months it will take to pay off the loan when you pay an extra \$250 per month

----- The amount of interest you save by paying an extra \$250 per month

## Project 4A: Baseball (Chapter 4)

A dominance relation matrix is a pre-cursor to transition matrices in chapter 9. You have a square matrix with the same labels for rows and columns. The value in any row and column represents the number of times the row player beat the column player.

For example, in the dominance relation shown in the table, Baltimore beat Boston 10 times, Boston beat New York 8 times, and Tampa Bay beat Toronto 9 times. The 0's on the main diagonal are because the teams never beat themselves (at least literally).

One use of the dominance relation matrix is in computing indirect winners. The idea being that if team A beats team B and team B beats team C, then team A is indirectly better than team C since they beat someone who beat team C.

|     | BAL | BOS | NYY | TBR | TOR |
|-----|-----|-----|-----|-----|-----|
| BAL | 0   | 10  | 7   | 8   | 12  |
| BOS | 9   | 0   | 8   | 11  | 13  |
| NYY | 12  | 11  | 0   | 12  | 9   |
| TBR | 11  | 8   | 7   | 0   | 9   |
| TOR | 7   | 6   | 10  | 10  | 0   |

Create a Google Spreadsheet for this project. Share it with the group members and the instructor. Create a dominance relation matrix for the 2017 National League Central Division of Major League Baseball and call it matrix A.

You can do this quickly by using the information at [baseball-reference.com](http://baseball-reference.com). Find a team, and then under the "Schedule & Results" for each team, they have a "Team Win/Loss Splits" that contains an "Opponent" table that lists how many times the team won or lost against the opponent. When gathering information, limit yourself to the National League Central Division teams.

For example, the 2017 Chicago Cubs information is at <http://www.baseball-reference.com/teams/CHC/2017-schedule-scores.shtml>

In an effort to predict the rankings for the 2018 season, a bookie awards points as follows: 15 points for each time the row team directly beats the column team, 7 points for each time the row team beat a team who beat the column team, and 3 points for each time the row team beat a team who beat a team who beat the column team.

----- Explain how matrices can be used to find the total points for each team.

----- What is the number of points the Brewers directly received from the Cubs?

----- What is the total number of points the Cubs received from the Reds?

----- List the teams and their overall scores. Who is the master beater (best team)?

## Project 4B: Networking (Chapter 4)

The incidence matrix is similar to the dominance relation matrix, except that the value in a row and column represent the number of direct routes from the row to the column.

Although they don't service Central Illinois, Cox is a major cable company. They have a map of their business network at

<http://ww2.cox.com/wcm/en/business/datasheet/national-ip-backbone-map.pdf>

Create a Google Spreadsheet for this project and share it with the group members.

Create an incidence matrix for the Cox business network, assuming that all traffic is bi-directional, between the regional data centers and call it matrix A. Ignore any local market backbone nodes. List all the cities alphabetically. A hop occurs every time a data packet goes from one center to another along a network segment.

*Be careful: You can get from Atlanta to Dallas 1) directly or by going through 2) Pensacola, 3) Macon, or 4) Gainesville so you would enter a 4 for that route. You would not include Atlanta to Baton Rouge to Dallas since Baton Rouge is a regional data center itself. The sum of each row should equal the number of paths leaving the regional data center except for weirdness like Baton Rouge or Kansas City that have loops going back to themselves without hitting going to another regional data center..*

----- The minimum number of hops needed to get from Omaha to Orange County

----- The regional data centers that are the furthest apart network-wise. That is, which sites require the most segments to get between them?

----- The minimum number of hops needed to guarantee a packet can travel from any data center to any other data center

----- The mathematical expression used to find the previous two answers.

Part of designing a good network is building in redundancy. If a regional center goes offline, the traffic needs to be able to get from it source to its destination, although it may need additional hops to do so.

----- The minimum number of hops needed to guarantee a packet can travel from any data to any other data center if Kansas City were to go offline.

## Project 8: Decision Theory (Chapter 8)

John and Mitchy run a computer store.

They can purchase 10 computers from Zol and Denny for \$1400 each, 30 computers from McGuinn and McGuire for \$1300 each, or 50 computers from Sebastian for \$1250 each (they can buy from more than one dealer, but only one order per dealer).

John and Mitchy sell the computers for \$1500 each. Each computer that is left at the end of the month will be sold in a clearance sale for \$900. John and Mitchy estimate a loss of goodwill of \$50 for each customer which comes into the store, but is unable to purchase a computer.

During the month, the customers will either demand 15, 30, 45, or 60 computers. Assume the probability of 15, 30, 45, or 60 computers is 0.10, 0.15, 0.50, and 0.25 respectively.

----- Create a payoff table with the five actions (remember that you can combine purchases from more than one dealer and some plans don't make any sense when the demand is considered) and four states of nature (demand)

| Payoff Table | Buy 10 | Buy 30 | Buy 40 | Buy 50 | Buy 60 |
|--------------|--------|--------|--------|--------|--------|
| Demand 15    |        |        |        |        |        |
| Demand 30    |        |        |        |        |        |
| Demand 45    |        |        |        |        |        |
| Demand 60    |        |        |        |        |        |

----- Create the opportunistic loss (regret) table.

----- For each decision criteria (expected value, maximax, maximin, minimax), find the payoff or loss for each action and the best action.

| Criterion  | Buy 10 | Buy 30 | Buy 40 | Buy 50 | Buy 60 | Best Action |
|------------|--------|--------|--------|--------|--------|-------------|
| Exp. Value |        |        |        |        |        |             |
| Maximax    |        |        |        |        |        |             |
| Maximin    |        |        |        |        |        |             |
| Minimax    |        |        |        |        |        |             |

## Project 9A: Airline Loyalty (Chapter 9)

The Central Illinois Regional Airport (CIRA) in Bloomington-Normal is served by American Airlines, Delta, and Allegiant. Of these, only American Airlines and Delta have daily flight service, so we will consider only those two airlines in this analysis.

Trip Advisor has airline recommendations and ratings at <http://www.tripadvisor.com/Airlines>. Whether or not a person would recommend an airline is implicitly determined from the overall rating for that airline. For example, if an airline has a rating of 3.5 out of 5 stars, we will take that to mean that  $3.5/5.0 = 0.70$  or 70% of people would recommend that airline.

Assume that if a customer would recommend an airline to someone that they will fly that airline again the next time they fly. If they would not recommend an airline to someone, then they will fly a competitor airline the next time they fly. Cost or destination is not a factor in choosing an airline.

John has recently started a new job in Bloomington that will require him to fly often. He has no previous experience flying, so he looks at the number of ratings each airline has received on Trip Advisor and randomly chooses an airline based on the relative frequency of ratings.

Example: If American has received 250 ratings and Delta has received 175 ratings, then there is a  $250/425$  chance John will pick American and a  $175/425$  chance he will pick Delta for the initial flight.

----- Draw a transition diagram and give the transition matrix for John's flights out of CIRA.

----- What is the probability that John's initial flight will be with Delta?

----- What is the probability that John's fourth flight will be with American Airlines?

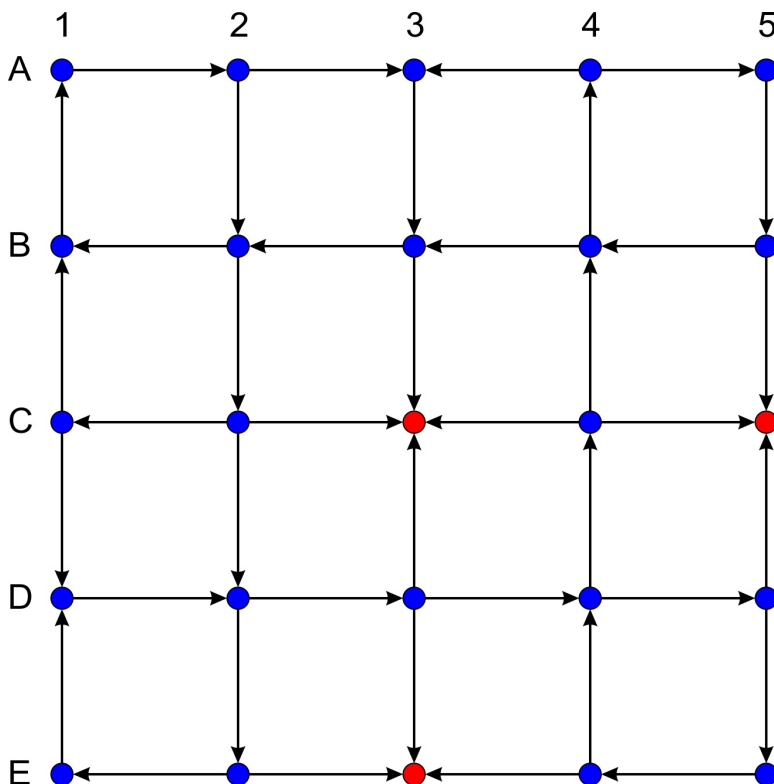
----- What is the long term probability that John will fly with Delta?

## Project 9B: Maze Movement (Chapter 9)

A 5×5 grid is laid out as shown in the figure.

Each node is labeled using a letter A-E for the row and a number 1-5 for the column. The top-left node would be A1, the bottom-right would be E5, while the center would be C3.

The arrows indicate the direction of movement between nodes. Movement continues until reaching a node with no exit (shaded red in the diagram).



- Create a transition matrix in standard form.
- Find the fundamental matrix  $F$ .
- If you start at node C1, how many transitions can be expected before ending?
- If you start at node E5, how many transitions can be expected before ending?
- If you start at node A2, what is the probability of eventually ending up in cell C5?
- If the object is to take as many turns as possible, at which node(s) should you start?
- If the object is to end at cell C3, in which transient node(s) should you start to maximize your chance?



## Project 10: Competing Stores (Chapter 10)

Rick and Corissa own the only two grocery stores in town. This means that a sale for Rick is a loss for Corissa and vice versa. Each week, they each run a special on exactly one type of food in an effort to draw business into their store. The matrix showing the choices and the gain in sales for Rick's store are shown.

|                   |         | Corissa's Country Market |       |        |       |
|-------------------|---------|--------------------------|-------|--------|-------|
|                   |         | Cereal                   | Dairy | Health | Meats |
| Rick's Ready Mart | Baking  | -3                       | 1     | -5     | 4     |
|                   | Fruits  | 2                        | -1    | -2     | 3     |
|                   | Pasta   | -1                       | 1     | 3      | -2    |
|                   | Seafood | 3                        | 4     | -1     | 2     |

Answer the following questions.

----- If Rick and Corissa each randomly select a food type to put on sale, what are the strategies and what is the value of the game for Rick?

----- What are the optimal strategies for Rick and Corissa? What is the value of the game for Rick under that those strategies?

----- Find the expected payoff values for each of Rick's actions if Corissa plays her optimal strategies. Find the expected loss values for each of Corissa's actions if Rick plays his optimal strategies.

Rick finds out that Corissa is going to spin the spinner from the game Life (10 slots) and place Cereal on sale if a 1 shows up, Dairy on sale if a 2 or 3 shows up, Health on sale if a 4, 5, or 6 shows up, and Meats on sale if a 7, 8, 9, or 10 shows up.

----- What is the expected value of each action for Rick?

----- What should Rick's *a priori* strategy be using the expected value criterion? *a priori* means with prior knowledge – that is, if Rick knows what Corissa is going to do, then what should Rick do?

----- What should Corissa's strategy really be (not what she said) if the intent was to trick Rick into playing a particular strategy?