Math 160 Projects

This course has some projects that require you to go beyond the level of material covered in the book or use outside sources to acquire data.

Although you may work the problems individually, the projects are designed to be group projects. They require extra work and you can benefit from the group. The exams will often see questions similar to those on the projects and so having other people verify the correctness of the problems before you take the exam can be beneficial.

Groups may have up to 3 people in them (except for the project 3A, which must be worked individually). You will be able to select your own groups for the projects and you may with each unit. Most submissions in Canvas ask you to identify who you worked with on the projects, even if you worked alone.

Some of these projects are very similar to problems that will appear on your exams. For that reason, it is important that you complete and understand them before taking your exam, even if they are not due until after the exam.

The questions that you need to answer are numbered in the projects. Projects will be turned in electronically and so make sure you answer those questions in the projects before you submit them.

The assignments in Canvas contain links to templates and forms for completing your projects. Some of those templates have one page for the work and another page for answers; others have work at the top and answers at the bottom. Make sure that you answer the questions.

Completed projects must be submitted using Canvas. Although you may work in groups, each person must individually submit their answers. You should leave a submission comment letting the instructor know who you worked with or that you worked alone.

Project 4A: Baseball (Ch 4)

The Canvas project page contains a link to a template that you should use for this project.

A dominance relation matrix is a pre-cursor to transition matrices in chapter 9. You have a square matrix with the same labels for rows and columns. The value in any row and column represents the number of times the row player beat the column player.

For example, in the dominance relation shown in the table, Baltimore beat Boston 10 times, Boston beat New York 8 times, and Tampa Bay beat Toronto 9 times. The 0's on the main diagonal are because the teams never beat themselves (at least literally).

Example: 2017 American League East

	BAL	BOS	NYY	TBR	TOR
BAL	0	10	7	8	12
BOS	9	0	8	11	13
NYY	12	11	0	12	9
TBR	11	8	7	0	9
TOR	7	6	10	10	0

One use of the dominance relation matrix is in computing indirect winners. The idea being that if team A beats team B and team B beats team C, then team A is indirectly better than team C since they beat someone who beat team C.

Create a dominance relation matrix for the **2022 National League Central Division** of Major League Baseball and call it matrix A.

You can do this quickly by using the information at baseball-reference.com. Find a team, and then under the "Schedule & Results" for each team, they have a "Team Win/Loss Splits" that contains an "Opponent" column that lists how many times the team won or lost against the opponent (you may need to expand the column to see all the teams). When gathering information, limit yourself to the National League Central Division teams.

For example, the 2022 Chicago Cubs information is at https://www.baseball-reference.com/teams/CHC/2022-schedule-scores.shtml

In an effort to predict the rankings for the 2023 season, a bookie awards points as follows: 3 points for each time the row team directly beats the column team, 2 points for each time the row team beat a team who beat the column team, and 1 point for each time the row team beat a team who beat a team who beat the column team.

1. Give an expression involving matrix A that computes the total points for each team.
2. What is the number of points the Brewers directly received from the Cubs?
3. What is the total number of points the Cubs received from the Reds?
4. List the teams and their overall scores. Which is the best team (scored most points)?

Project 4B: Networking (Ch 4)

The Canvas project page contains a link to a template that you should use for this project.

The incidence matrix is similar to the dominance relation matrix, except that the value in a row and column represent the number of direct routes from the row to the column.

The Internet2 is an internet provider for research and education institutions. Their members include University of Illinois at Urbana-Champaign and Instructure (the company that makes the Canvas learning management system). Here is a link to their network as of March 2022, but you may want to access it through the project page in Canvas: https://storage.pardot.com/66332/164745521835nC3kYg/NS_map_march22.png

Create an incidence matrix for the Internet 2 backbone, assuming that all traffic is bi-directional, between the cities where three or more connetions exist and call it matrix **A**. Ignore any other cities. A *hop* occurs every time a data packet goes from one center to another along a network segment. The values that go in the incidence matrix are the number of direct connections between those two cities. For most cities, this will either be 0 (there is no direct route) or 1 (there is 1 direct route).

Be careful: The city labeled Houston is actually Galveston. Houston is the city above that with the multiple connections. I've combined Ashburn, VA, and Washington, DC, together. There are no cities in the Northeast with more than 2 connections, which means that the loop through Boston is a second way that you can get from Cleveland to Ashburn, so you would enter 2 for the number of ways to get between those cities.

The sum of each row should equal the number of connections coming into each city.

1. What is the minimum number of hops needed to get from Houston to Sacramento?
2. What is the minimum number of hops needed to guarantee a packet can travel from any city to any other city?
3. Which matrix (write the answer in terms of matrix **A**) did you look at to find the answer to the previous question?
4. Part of designing a good network is building in redundancy. If a city loses connection, the traffic needs to be able to get from it source to its destination, although it may need additional hops to do so. Answer this question when Chicago loses connection.
    (a) What is the minimum number of hops needed to guarantee a packet can travel between any two cities if Chicago were to lose connection?
    (b) Which two cities that are the furthest apart network-wise when Chicago loses connection? That is, which sites require the most hops to get between them?

Project 5: Graphical Solutions (Ch 5)

The Canvas project page contains a link to a template that you should use for this project.

Consider the following linear programming problem.

$$\begin{array}{lrcrcrcr} \text{Maximize} & P & = & 3x & + & 4y & & \\ \text{Subject to} & & & x & + & 2y & \leq & 12 \\ & & & x & + & y & \leq & 8 \\ & & & 2x & + & 3y & \geq & 12 \\ & & & x & , & y & \geq & 0 \end{array}$$

1. Use the Desmos Graphing Calculator at https://www.desmos.com/calculator to graph the feasible region.

   - Negate each inequality when graphing so that the unshaded portion is the feasible region.
   - Add each corner point to the graph and label the coordinates.
   - Zoom in tightly to the feasible region. Do not worry about keeping the x-scale and y-scale the same.

   When done, click on Share and then Export Image. Set the size to Medium Square with Medium lines. Then download the PNG file. Insert the image into your document.

2. Complete the table of corner points and find the value of the objective function for each corner point. To label the corner points, start at the origin and move clockwise (first moving upwards) around the region. Let A be the first corner point, B be the second corner point, and so on.

Pt	x	y	P
A			
B			
C			
D			
E			

3. Highlight the row of the table that contains the solution.

Project 6: College Basketball (Ch 6)

The Canvas project page contains a link to data that you should use for this project.

An internet gambling company feels that there is a lot of money to be made from the teams that didn't make the NCAA March Madness. They decided to take the top player from the two worst teams in each of the 32 NCAA Division 1 basketball conferences and organize an alternative tournament called "April Antics".

To avoid lawsuits from the NCAA, they decide to organize the tournament very differently than March Madness. The 64 players will be divided into nine teams. Those nine teams will be divided into three round-robin tournaments of three teams. The winner of each round-robin tournament will advance to a final round-robin tournament of three teams, where the winning team is the champion.

The gambling company realizes there is more money to be made when it is harder for people to predict the winner, so the new system tries to balance the teams so they are similar. One technique of measuring similarity is to use the standard deviation between the values. The less variation there is in the values, the smaller the standard deviation will be. Mathematically, variances are better when adding values together than standard deviations. Since we will have multiple measures to balance, we will create a composite score by adding the variances together.

Here is how the process will work.

1. Randomly assign players to one of 9 teams.
2. Find the average for each measure for the teams in the bracket.
3. Find the sample variance for each of the 9 averages for each measure.
4. Minimize the sum of the variances.

The measures used are:

- **mpg**: the average minutes played per game for the player
- **per**: the player efficiency rating
- **tsp**: the true shooting percentage of the player
- **usage**: the percentage of the plays involving the player when he was on the floor

The following rules must also be followed.

- There are nine teams
- Each team must have at least six players.

1. Use the evolutionary method of Excel's solver to find the best way of assigning players to teams.
2. Sort the list by team, school, and player.
3. Submit the Excel spreadsheet with the Solver setup (do not copy the players to a new spreadsheet when done).

Project 8: Decision Theory (Ch 8)

The Canvas project page contains a link to a template that you should use for this project.

Tom runs a computer store.

He can purchase 10 computers from Quanta for $850 each, 30 computers from Compal for $800 each, or 50 computers from Wistron for $750 each. He can order from more than one manufacturer, but only one order from each.

During the month, Tom sells the computers for $1100 each. Any unsold computers at the end of the month are sold (for sure) for $600. Tom estimate a loss of goodwill of $100 for each customer which comes into the store, but is unable to purchase a computer.

He estimates the monthly demand will be 15, 30, 45, or 60 computers with probabilities of 0.1, 0.4, 0.3, and 0.2 respectively.

1. Create a payoff table with the five actions (remember that you can combine purchases from more than one dealer and some plans don't make any sense when the demand is considered) and four states of nature (demand)
2. Create the opportunistic loss (regret) table.
3. For each decision criteria (expected value, maximax, maximin, minimax), find the payoff or loss for each action and the best action.

Payoff Table

	Buy 10	Buy 30	Buy 40	Buy 50	Buy 60
Demand 15					
Demand 30					
Demand 45					
Demand 60					

Opportunistic Loss Table

	Buy 10	Buy 30	Buy 40	Buy 50	Buy 60
Demand 15					
Demand 30					
Demand 45					
Demand 60					

Actions

Criterion	Buy 10	Buy 30	Buy 40	Buy 50	Buy 60	Best Action
Exp. Value						
Maximax						
Maximin						
Minimax						

Project 9A: Airline Loyalty (Ch 9)

The Canvas project page contains a link to a template that you should use for this project.

The Central Illinois Regional Airport (CIRA) in Bloomington-Normal is served by American Airlines, Delta, and Allegiant. Of these, only American Airlines and Delta have daily flight service, so we will consider only those two airlines in this analysis.

Trip Advisor has airline recommendations and ratings at https://www.tripadvisor.com/Airlines.

Whether or not a person would recommend an airline is implicitly determined from the overall rating for that airline. For example, if an airline has a rating of 3.5 out of 5 stars, we will take that to mean that 3.5/5.0 = 0.70 or 70% of people would recommend that airline.

Assume that if a customer would recommend an airline to someone that they will fly that airline again the next time they fly. If they would not recommend an airline to someone, then they will fly a competitor airline the next time they fly. Cost or destination is not a factor in choosing an airline.

John has recently started a new job in Bloomington that will require him to fly often. He has no previous experience flying, so he looks at the number of ratings each airline has received on Trip Advisor and randomly chooses an airline based on the relative frequency of ratings.

To help you understand, pretend that American has received 250 ratings and Delta has received 175 ratings, then there are 425 ratings all together. There is a 250/425 chance John will pick American and a 175/425 chance he will pick Delta for the initial flight.

Do not round any of the values until you are done with the calculations (use 250/425, not 0.59). Give at least four decimals in your answers.

1. Draw a transition diagram and give the transition matrix for John's flights out of CIRA.
2. What is the probability that John's initial flight will be with Delta?
3. What is the probability that John's fourth flight will be with American Airlines?
4. What is the long term probability that John will fly with Delta?

Project 9B: Maze Movement (Ch 9)

The Canvas project page contains a link to a template that you should use for this project.

A 5×5 grid is laid out as shown in the figure. Each node is labeled using a letter A-E for the row and a number 1-5 for the column. The arrows indicate the direction of movement between nodes.

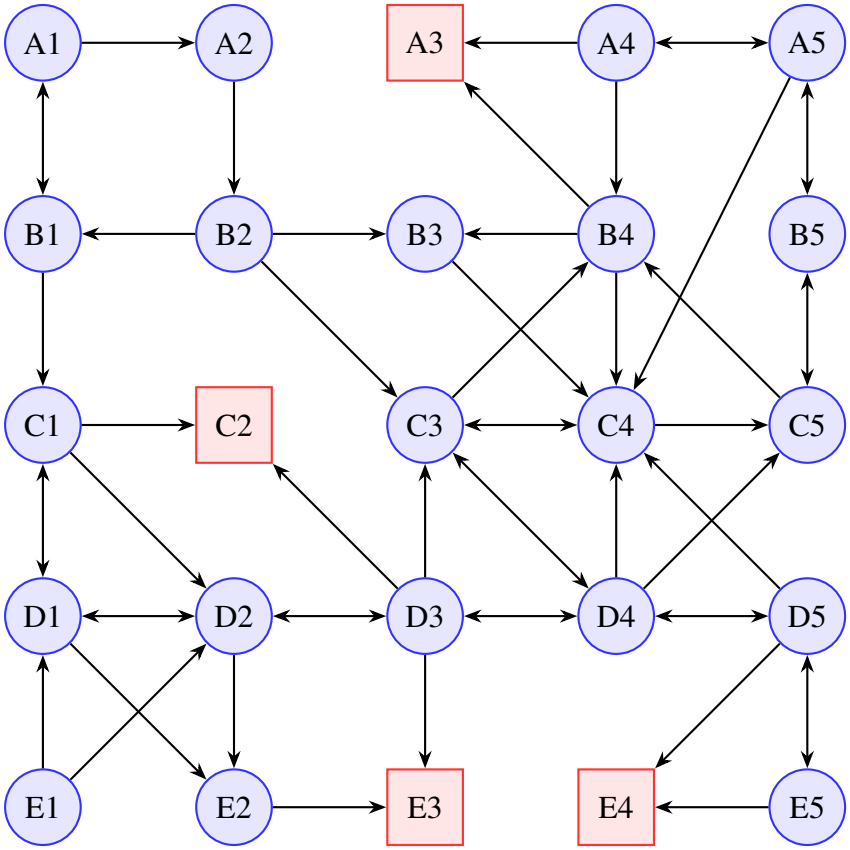
You begin at a node and then, with each transition, either move from a blue circle to a connected node or remain at a red square.

When moving, you randomly select, with equal probability, a node from any of the connected nodes. You may not remain in the current node if you have the possibility of moving. The probability of moving is determined by the number of adjacent nodes.

For example, if you are at node C3, you can move to nodes B4, C4, or D4, so each would have a probability of 1/3. But if you were at node C4, then you could move to nodes C3 or C5, so the probability of each would be 1/2.

Movement continues until reaching a node with no exit (a square with a red background).

The arrows are important! You can go from A1 to A2 but not A2 to A1.



1. Create a transition matrix in standard form.
2. Find the fundamental matrix F.
3. If you start at node D2, how many times can you expect to spend in node C3?
4. If you start at node C1, how many transitions can be expected before ending?
5. If you start at node C4, how many transitions can be expected before ending?
6. If you start at node A2, what is the probability of eventually ending at node E3?
7. If the object is to make as many transitions as possible before reaching an absorbing node, at which node(s) should you start?
8. If the object is to end at node A3, in which non-adjacent transient node(s) should you start to maximize your chance?

Project 10: Competing Stores (Ch 10)

The Canvas project page contains a link to a template that you should use for this project.

Rick and Corissa own the only two grocery stores in town. This means that a sale for Rick is a loss for Corissa and vice versa.

Each week, they each run a special on exactly one type of food in an effort to draw business into their store. The matrix showing the choices and the gain in sales for Rick's store is provided.

	Cereal	Dairy	Health	Meats
Baking	2	−1	−3	0
Fruits	−3	4	1	−2
Pasta	−1	0	−2	2
Seafood	3	−2	2	−3

Rick, the row player, runs specials on baking, fruits, pasta, or seafood.

Corissa, the column player, runs specials on cereal, dairy, health, and meats.

Answer the following questions. Give all answers as fractions!

1. If Rick and Corissa each randomly select a food type to put on sale, what are the strategies and what is the value of the game for Rick?

2. What are the optimal strategies for Rick and Corissa? What is the value of the game for Rick under those strategies?

3. Find the expected payoff values for each of Rick's actions if Corissa plays her optimal strategies. Find the expected loss values for each of Corissa's actions if Rick plays his optimal strategies.

4. Rick conducts some espionage and finds out that Corissa is going to spin the spinner from the game Life (the spinner has the numbers from 1 to 10 and each is equally likely).

   He learns that Corissa will place cereal on sale if the spinner lands on a 1, dairy if it lands on a 2 or 3, health if it lands on 4, 5, or 6, and meats if it lands on a 7, 8, 9, or 10.

   (a) If Corissa plays uses this strategy, what is the expected value of each action for Rick?

   (b) What should Rick's *a priori* strategy be using the expected value criterion? *a priori* means with prior knowledge – that is, if Rick knows what Corissa is going to do, then what should Rick do?

   (c) What should Corissa's strategy really be (not what she said) if she knew he had spies and the intent was to trick Rick into playing a particular strategy?

5. **Decision Theory**: Find the best actions for Rick under the expected value, maximax, maximin, and minimax criteria. Use the appropriate optimal strategy when finding the expected values.

Actions				
Criterion	Exp. Value	Maximax	Maximin	Minimax
Baking				
Fruits				
Pasta				
Seafood				
Best Action				

Project 3A: Planning for Retirement (Ch 3)

Your answers for this project are entered into a Canvas quiz.

This project is to designed to help you get a sense of what is needed to retire. When estimating costs, remember that inflation will occur and things will be more expensive than they are now, but that by the that time , you are likely to have most major purchases like a house or car already purchased.

All ages should be in years, and for simplicity, we will assume that all major events (start saving, retiring, and dying) occur on your birthday so that we don't have to mess with partial years. For purposes of this project, you must not plan on retiring for at least 15 years. Round all dollar amounts to the nearest cent. Assume that all money is invested and earns interest at an annual nominal rate of 9%, compounded monthly.

This is an individual project. You may help each other, but the numbers need to be yours.

1. Age, in years, that you will be on your next birthday
2. Age, in years, when you plan on retiring and
3. years until retirement
4. Number of years that you expect to live and
5. years of retirement
6. Amount, in dollars, that you anticipate needing each month during your retirement
7. Amount of money that you need to have saved the day you retired in order to fund your retirement
8. Amount of money that you will need to save each month until you retire in order for you to reach the amount needed to retire
9. Amount of money you will have after 10 years of regular payments into your retirement fund
10. After 10 years, a trust fund established upon the death of your grandparents gives you a one-time payment of $150,000. You spend $50,000 of it to pay off some college and credit card debts and put the rest of it into your retirement fund and **continue making your regular deposits** until you retire.
    (a) Amount of money that the $100,000 from the trust fund will grow to before you retire
    (b) Amount of money you'll be able to spend each month during retirement when you include the investment results from the trust fund

Project 3B: Buying a Home (Ch 3)

Your answers for this project are entered into a Canvas quiz.

The purpose of this project is to help you understand purchasing a home. You will make a 20% down payment to avoid escrow costs and finance the balance over 30 years with a fixed loan rate of 7.5%. Payments and compounding will occur monthly. Round all dollar amounts to the nearest cent.

Find a home in the Richland district that is between $100,000 and $250,000 by visiting https://www.realtor.com, https://www.zillow.com or some other online real-estate system.

1. Address of home
2. Asking price of home
3. Amount financed after 20% down payment
4. Monthly payment needed to finance home
5. Total cost to repay loan if regular payments are made for the full 30 years (the loan does not include the down payment)
6. Amount of interest paid if the loan goes full term
7. For the next two questions, assume that ten years of regular payments have been made.
   (a) Amount still owed on the house
   (b) Equity in home if the house is appraised at 8% more than you originally paid for it
8. For the next two questions, assume that from the beginning of the loan, you have paid an extra $300 per month. This section has nothing to do with the 10 years in the previous two questions.
   (a) The number of months it will take to pay off the loan when you pay an extra $300 per month
   (b) The amount of interest you save by paying an extra $300 per month