

Common Maclaurin Series

Interval of
Convergence

$\frac{1}{1-x}$	$\sum_{k=0}^{\infty} x^k$	$1 + x + x^2 + x^3 + \dots$	$(-1, 1)$
$\frac{1}{1+x^2}$	$\sum_{k=0}^{\infty} (-1)^k x^{2k}$	$1 - x^2 + x^4 - x^6 + \dots$	$(-1, 1)$
e^x	$\sum_{k=0}^{\infty} \frac{x^k}{k!}$	$1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \dots$	$(-\infty, +\infty)$
$\sin x$	$\sum_{k=0}^{\infty} \frac{(-1)^k x^{2k+1}}{(2k+1)!}$	$x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots$	$(-\infty, +\infty)$
$\cos x$	$\sum_{k=0}^{\infty} \frac{(-1)^k x^{2k}}{(2k)!}$	$1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots$	$(-\infty, +\infty)$
$\ln(1+x)$	$\sum_{k=1}^{\infty} \frac{(-1)^{k+1} x^k}{k}$	$x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots$	$(-1, 1]$
$\tan^{-1} x$	$\sum_{k=0}^{\infty} \frac{(-1)^k x^{2k+1}}{2k+1}$	$x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \dots$	$[-1, 1]$
$\sinh x$	$\sum_{k=0}^{\infty} \frac{x^{2k+1}}{(2k+1)!}$	$x + \frac{x^3}{3!} + \frac{x^5}{5!} + \frac{x^7}{7!} + \dots$	$(-\infty, +\infty)$
$\cosh x$	$\sum_{k=0}^{\infty} \frac{x^{2k}}{(2k)!}$	$1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \frac{x^6}{6!} + \dots$	$(-\infty, +\infty)$
$(1+x)^m$	$1 + \sum_{k=1}^{\infty} \frac{m(m-1)\dots(m-k+1)}{k!} x^k, m \neq 0, 1, 2, \dots$		$(-1, 1)^*$

* Note: The behavior at the endpoints depends on m : For $m > 0$, the series converges at both endpoints; for $m \leq -1$, the series diverges at both endpoints; and for $-1 < m < 0$, the series converges conditionally at $x = 1$ and diverges at $x = -1$.

Convergence Tests

Name	Summary
Divergence Test	If the terms of the sequence don't go to zero, the series diverges.
Integral Test	The series and the integral do the same thing.
p-series	Series converges if $p > 1$.
Geometric Series	The series converges if the absolute value of the common ratio is less than 1.
Direct Comparison Test	If the larger series converges, so does the smaller. If the smaller series diverges, so does the larger.
Limit Comparison Test	If the ratio of the sequences is positive and finite, then both series do the same thing.
Ratio Test	Find the ratio of two consecutive terms. If the ratio is less than 1, the series converges. If the ratio is greater than 1, the series diverges.
Root Test	Take the n-th root of the sequence. If the ratio is less than 1, the series converges. If the ratio is greater than 1, the series diverges.
Alternating Series Test	If the sequence alternates, the terms in the sequence are decreasing, and approaching 0, then the series converges.

Where the word *sequence* is used, it refers to the terms inside the summation.