Skills Needed for Success in Calculus 1

There is much apprehension from students taking Calculus. It seems that for many people, "Calculus" is synonymous with "difficult." However, any teacher of Calculus will tell you that the reason that students are not successful in Calculus is not because of the Calculus, it's because their algebra and trigonometry skills are weak. You see, Calculus is really just one additional step beyond algebra and trig. Calculus is algebra and trigonometry with limits and limits aren't really that hard once you figure them out. There is often only one step in the problem that actually involves calculus, the rest is simplifying using algebra and trigonometry. That's why it is crucial that you have a good background in those subjects to be successful in calculus.

More good news about calculus is that we live in the real world, we don't deal with imaginary numbers (except for section 9.4, which isn't in Calculus 1). Also, in Calculus 1, we don't deal with logarithmic or exponential functions, which seem to give some people great difficulty.

The purpose of this document is to help identify some of those areas where you will need good algebra and trigonometry skills so that your calculus experience can be successful, pleasant, and rewarding.

Algebra Skills Needed

Factoring
You need to be able to factor expressions and equations like it was second nature to you. Many of the problems in calculus will involve finding the roots of a function and for the most part that means factoring. Don't just concentrate on polynomial factoring, either; you need to be able to factor expressions with rational exponents.

Here is an example of factoring out the greatest common factor, which is involves taking the smallest exponent on all of the common terms.

$$5x^2(2x-3)^{1/3}(3x+2)^{1/2}+8x(2x-3)^{-2/3}(3x+2)^{3/2}$$
$$=x(2x-3)^{-2/3}(3x+2)^{1/2}\left[5x(2x-3)+8(3x+2)\right]$$
$$=x(2x-3)^{-2/3}(3x+2)^{1/2}\left[10x^2-15x+24x+16\right]$$
$$=x(2x-3)^{-2/3}(3x+2)^{1/2}\left(10x^2+9x+16\right)$$

Know how to recognize and factor the special patterns of the difference of two squares, the difference of two cubes, and the sum of two cubes. Know that the sum of two squares usually doesn't factor in the real world.

Difference of two squares: $x^2 - y^2 = (x-y)(x+y)$

Sum / difference of two cubes: $x^3 \pm y^3 = (x \pm y)(x^2 \mp xy + y^2)$

Completing the Square

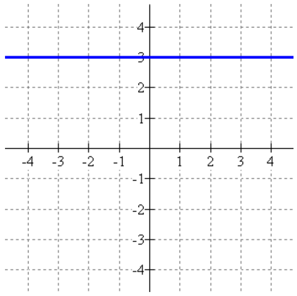
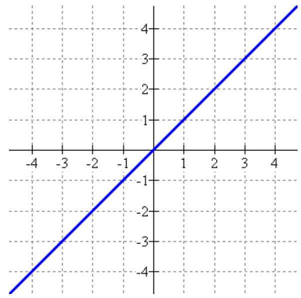
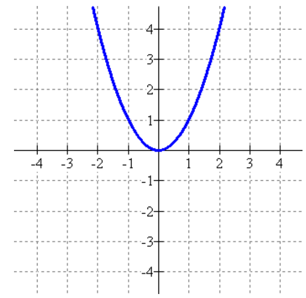
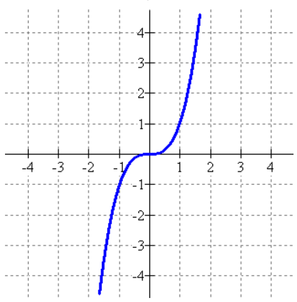
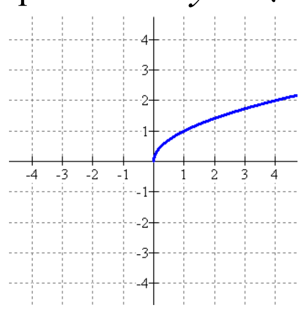
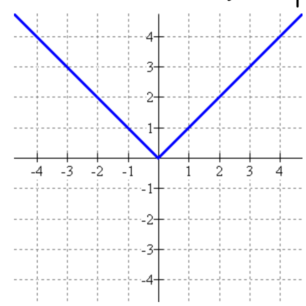
Another task that you will be called on to perform occasionally is completing the square. You need to be able to do this with both an equation and an expression. Examples of both are shown below.

Completing the square by adding to both sides of the equation	Completing the square by adding and subtracting on the same side
$x^2 + 3y^2 + 2x - 12y = 3$ $(x^2 + 2x) + 3(y^2 - 4y) = 3$ $(x^2 + 2x + 1) + 3(y^2 - 4y + 4) = 3 + 1 + 12$ $(x+1)^2 + 3(y-2)^2 = 16$	$f(x) = 5x - x^2$ $f(x) = -(x^2 - 5x)$ $f(x) = -\left(x^2 - 5x + \frac{25}{4}\right) + \frac{25}{4}$ $f(x) = -\left(x - \frac{5}{2}\right)^2 + \frac{25}{4}$

Basic Functions and Transformations

Algebra (and calculus) can be simplified if you understand that there are basic functions and that many of the other functions are transformations of those basic building blocks. You should be able to sketch the graph and know the domain and range of the basic functions upon sight.

You should also be able to recognize and apply transformations to the basic functions. Consider $y = 2 - (x-3)^2$. You should be able to recognize that the basic shape is the quadratic function $y = x^2$. To that basic shape, you have reflected it about the *x*-axis $y = -x^2$, shifted it up two units $y = 2 - x^2$, and shifted it right three units $y = 2 - (x-3)^2$.

Constant $y = k$  Domain: $(-\infty, +\infty)$ Range: $\{k\}$	Linear $y = x$  Domain: $(-\infty, +\infty)$ Range: $(-\infty, +\infty)$	Quadratic $y = x^2$  Domain: $(-\infty, +\infty)$ Range: $[0, \infty)$
Cubic $y = x^3$  Domain: $(-\infty, +\infty)$ Range: $(-\infty, +\infty)$	Square Root $y = \sqrt{x}$  Domain: $[0, \infty)$ Range: $[0, \infty)$	Absolute Value $y = \lvert x \rvert$  Domain: $(-\infty, +\infty)$ Range: $[0, \infty)$

Simplifying Expressions
Much of your time in this course will be spent simplifying the results of an expression that you obtained. Know how to combine similar or like terms and know the properties of exponents like adding exponents when multiplying factors that have the same base or multiplying exponents when raising to a power.

Formula Manipulation
You need to be able to work with formulas as well as have a good recall of basic geometry formulas for area and volume for common figures. There are geometric formulas on the inside front cover of your text as a resource.

Formula manipulation is much more than just memorizing formulas and plugging the values into them, however. You will need to solve for different variables and you will need to combine formulas together to come up with new formulas.

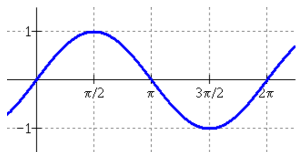
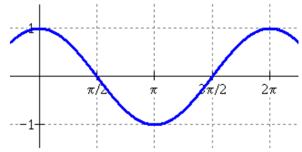
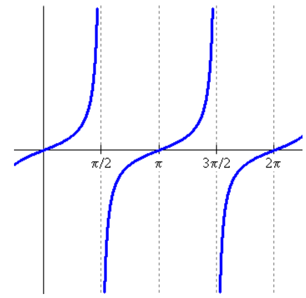
Example: A right circular cylinder has a volume of 10 cm^3 and its height is twice its circumference. Find the radius and height of the cylinder. The formula for the volume of any cylinder with parallel bases is $V = Bh$, where B is the area of the base. Since the base is a circle, the area of the base is $B = \pi r^2$. The circumference of a circle is $C = 2\pi r$ and in this cylinder, the height is twice the circumference, so the height is $h = 2C = 2(2\pi r) = 4\pi r$. The volume becomes $V = (\pi r^2)(4\pi r) = 4\pi^2 r^3$. Since we know that the volume is $V = 10$, we get $10 = 4\pi^2 r^3$. Solving that for r gives $r^3 = \dfrac{5}{2\pi^2}$ or $r = \sqrt[3]{\dfrac{5}{2\pi^2}}$ cm. The height is $h = 4\pi r = 4\pi\sqrt[3]{\dfrac{5}{2\pi^2}}$ cm, but that simplifies to be $h = \sqrt[3]{64\pi^3 \dfrac{5}{2\pi^2}} = 2\sqrt[3]{20\pi}$ cm.

Using Your Calculator
This may sound like a given condition by the time you get to calculus, but you need to be able to graph functions and get a proper viewing window. You should be able to use the Calc menu on your calculator to find roots, minimums, maximums, and intersections. You should also know how to use the table mode on your calculator. You should also know how to change the mode on your calculator and leave it in Radian mode for most of this course.

Trigonometry Skills Needed

Appendix A in the textbook contains a review of Trigonometry. You really need to know everything in it with the exception of the product to sum and sum to product formulas (formulas 47-53). Most of this will need to be memorized so that it is available for instant recall. Other things can be derived by understanding the relationships between the trigonometric functions and the different quadrants.

You should be able to sketch the basic trigonometric functions and be able to apply transformations to them. For example, consider the function $y = 3\sin(2x-1)+5$. You should be able to pick out that the basic graph is the sine function $y = \sin x$, that the amplitude is 3 because of $y = 3\sin x$, the period is $\dfrac{2\pi}{2} = \pi$ from $y = 3\sin(2x)$, the phase shift is $\frac{1}{2}$ unit to the right $y = 3\sin\left[2\left(x-\frac{1}{2}\right)\right]$, and the entire graph has been shift up five units $y = 3\sin\left[2\left(x-\frac{1}{2}\right)\right]+5$.

Sine $y = \sin x$	Cosine $y = \cos x$	Tangent $y = \tan x$
		
Domain: $(-\infty, +\infty)$	Domain: $(-\infty, +\infty)$	Domain: $x \neq \frac{\pi}{2} \pm k\pi$
Range: $[-1, 1]$	Range: $[-1, 1]$	Range: $(-\infty, +\infty)$
Period: 2π	Period: 2π	Period: π

Memorize the values of the three trigonometric functions for the special angles!

In this class, you will be expected to give exact answers in most cases. That means writing $1+\sqrt{2}$ instead of 2.414 or $\frac{17}{3}\pi$ and not 17.802. The good news is that you will not usually have to rationalize your denominators and writing $\frac{1}{\sqrt{2}}$ is okay.