Math 121: Mathematical Notation

Purpose:
One goal in any course is to properly use the language of that subject. Calculus is no different and may often seem like a foreign language. These notations summarize some of the major concepts and more difficult topics of the unit. Typing them helps you learn the material while teaching you to properly express mathematics on the computer. Part of your grade is for *properly* using mathematical content.

Instructions:
Use Word or WordPerfect to recreate the following documents. Each article is worth 10 points and should be emailed to the instructor at james@richland.edu. This is not a group assignment, each person needs to create and submit their own notation.

Type your name at the top of each document. Include the title as part of what you type. The lines around the title aren't that important, but if you will type ----- at the beginning of a line and hit enter, both Word and WordPerfect will convert it to a line.

For expressions or equations, you should use the equation editor in Word or WordPerfect. The instructor used WordPerfect and a 14 pt Times New Roman font with 0.75" margins, so they may not look exactly the same as your document.

If there is an equation, put both sides of the equation into the same equation editor box instead of creating two objects. Be sure to use the proper symbols, there are some instances where more than one symbol may look the same, but they have different meanings and don't appear the same as what's on the assignment. There are some useful tips on the website at http://people.richland.edu/james/editor/

If you fail to type your name on the document, you will lose 1 point. Don't type the hints or reminders that appear on the pages.

These notations are due two class periods after we finish the lecture for that material. Due dates are listed on the course calendar. Late work will be accepted but will lose 20% of its value per class period. If I receive your emailed assignment more than one class period before it is due and you don't receive all 10 points, then I will email you back with things to correct so that you can get all the points. Any corrections need to be submitted by the due date and time or the original score will be used.

Chapter 1 - Trigonometry Review

Degrees	$0°$	$30°$	$45°$	$60°$	$90°$
Radians	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$
$\sin\theta$	$\sqrt{0}/2=0$	$\sqrt{1}/2=1/2$	$\sqrt{2}/2$	$\sqrt{3}/2$	$\sqrt{4}/2=1$
$\cos\theta$	$\sqrt{4}/2=1$	$\sqrt{3}/2$	$\sqrt{2}/2$	$\sqrt{1}/2=1/2$	$\sqrt{0}/2=0$
$\tan\theta$	0	$1/\sqrt{3}$	1	$\sqrt{3}$	undefined

An angle θ in QI becomes $\pi-\theta$ in QII, $\pi+\theta$ in QIII, and $2\pi-\theta$ or $-\theta$ in QIV.

$\sin(-x)=-\sin x \qquad \cos(-x)=\cos x \qquad \tan(-x)=-\tan x$

$\sin^2 x+\cos^2 x=1 \quad 1+\tan^2 x=\sec^2 x \quad \cot^2 x+1=\csc^2 x$

$$\sin\left(\frac{\pi}{2}-x\right)=\cos x \quad \cos\left(\frac{\pi}{2}-x\right)=\sin x \quad \tan\left(\frac{\pi}{2}-x\right)=\cot x$$

$$\cos(a\pm b)=\cos a\cos b\mp\sin a\sin b$$

$$\sin(a\pm b)=\sin a\cos b\pm\cos a\sin b$$

$$\tan(a\pm b)=\frac{\tan a\pm\tan b}{1\mp\tan a\tan b}$$

$$\cos 2x=\cos^2 x-\sin^2 x \quad \sin 2x=2\sin x\cos x \quad \tan 2x=\frac{2\tan x}{1-\tan^2 x}$$

$$\cos^2 x=\frac{1+\cos 2x}{2} \quad \sin^2 x=\frac{1-\cos 2x}{2}$$

Chapter 2 - Limits

When finding a finite limit, simply substitute the value into the expression unless it causes problems.

The two sided limit $\lim\limits_{x \to a} f(x)$ exists if and only if both one sided limits $\lim\limits_{x \to a^+} f(x)$ and $\lim\limits_{x \to a^-} f(x)$ exist and are equal to each other.

If a rational function has a limit of the form 0/0, then there is a common factor in both the numerator and the denominator. Factor both, reduce, and then evaluate the limit.

When finding infinite limits of polynomial and rational functions, only the leading term needs to be considered. This is only true for limits as $x \to +\infty$ or $x \to -\infty$.

$$\lim_{x \to \infty}\left(a_n x^n + a_{n-1}x^{n-1} + \cdots + a_0\right) = \lim_{x \to \infty} a_n x^n$$

$$\lim_{x \to \infty}\left(\frac{a_n x^n + a_{n-1}x^{n-1} + \cdots + a_0}{b_m x^m + b_{m-1}x^{m-1} + \cdots + b_0}\right) = \lim_{x \to \infty} \frac{a_n x^n}{b_m x^m}$$

The Definition of a Limit

$\lim\limits_{x \to a} f(x) = L$ if $\forall \varepsilon > 0,\ \exists \delta > 0 \ni \left| f(x) - L \right| < \varepsilon$ whenever $0 < \left| x - a \right| < \delta$.

The essence is that you need to find δ by turning $\left| f(x) - L \right| < \varepsilon$ into $\left| x - a \right| < \delta$.

Common Trigonometric Limits

$$\lim_{x \to 0} \frac{\sin x}{x} = 1 \quad \lim_{x \to 0} \frac{1 - \cos x}{x} = 0 \quad \lim_{x \to 0} \frac{\tan x}{x} = 1$$

A function *f* is **continuous** at $x = a$ if 1) $f(a)$ is defined, 2) $\lim\limits_{x \to a} f(x)$ exists, and 3) $\lim\limits_{x \to a} f(x) = f(a)$. If the last condition is satisfied, so are the first two.

Chapter 3 - Derivatives

First derivatives: $\frac{d}{dx}\left[f(x)\right]=f'(x)=D_x\left[f(x)\right]=\frac{dy}{dx}=y'$

Second derivatives: $\frac{d^2}{dx^2}\left[f(x)\right]=f''(x)=D_{xx}\left[f(x)\right]=\frac{d^2y}{dx^2}=y''$

Limit definition of a derivative: $f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$

The **power rule for derivatives** is $\frac{d}{dx}\left[x^n\right]=n\cdot x^{n-1}$, which says multiply by the exponent and then subtract one from the exponent.

The **product rule** says to find the sum of the terms where you take the derivative of one factor at a time : $(fg)'=f'g+fg'$ or $(fgh)'=f'gh+fg'h+fgh'$.

The **quotient rule** is $\left(\frac{f}{g}\right)'=\frac{gf'-fg'}{g^2}$, which can be sung as "lo d-hi minus hi d-low, square the bottom and there you go."

The **chain rule** is $\left[f(g(x))\right]'=f'(g(x))\cdot g'(x)$ or $\frac{dy}{dx}=\frac{dy}{du}\cdot\frac{du}{dx}$, which says to keep multiplying by the derivatives of the inside function.

Derivatives of Trigonometric Functions

$\frac{d}{dx}[\sin x]=\cos x$ $\quad$ $\frac{d}{dx}[\tan x]=\sec^2 x$ $\quad$ $\frac{d}{dx}[\sec x]=\sec x\tan x$

$\frac{d}{dx}[\cos x]=-\sin x$ $\quad$ $\frac{d}{dx}[\cot x]=-\csc^2 x$ $\quad$ $\frac{d}{dx}[\csc x]=-\csc x\cot x$

The **Local Linear Approximation** is $f(x)\approx f(x_0)+f'(x_0)\cdot\Delta x$

Chapter 4 - Applications of the Derivative

If f is differentiable, then f is **increasing** when $f'(x) > 0$, **decreasing** when $f'(x) < 0$, and **constant** when $f'(x) = 0$.

Critical points occur where $f'(x) = 0$ or $f'(x)$ is undefined. **Stationary points** are the critical points where $f'(x) = 0$.

If f is twice differentiable, then f is **concave up** when $f''(x) > 0$ and **concave down** when $f''(x) < 0$.

Inflection points occur when concavity changes. This can occur when $f''(x) = 0$ or $f''(x)$ is undefined.

Relative extrema (maximums or minimums) can only occur at critical points, however not every critical point is a relative extremum.

The **second derivative test** says: If f is twice differentiable at $x = a$ and $f'(a) = 0$, then there will be a relative minimum at $x = a$ if $f''(a) > 0$ and a relative maximum at $x = a$ if $f''(a) < 0$. If $f''(a) = 0$, the second derivative test is inconclusive.

Rectilinear Motion

Position	$s(t)$
Velocity	$v(t) = s'(t) = \dfrac{ds}{dt}$
Speed	$speed = \lvert v(t) \rvert = \left\lvert \dfrac{ds}{dt} \right\rvert$
Acceleration	$a(t) = v'(t) = \dfrac{dv}{dt} = s''(t) = \dfrac{d^2 s}{dt^2}$

Chapter 5 - Integration

$$\frac{d}{dx}\left[\int f(x)dx\right] = f(x)$$

The **power rule for integrals** is $\int x^n dx = \frac{1}{n+1}x^{n+1} + C$

The conversion between **Riemann sum** and definite integral is

$$\int_a^b f(x)dx = \lim_{\max \Delta x_k \to 0} \sum_{k=1}^{n} f\left(x_k^*\right)\Delta x_k$$

Properties of Definite Integrals

$$\int_a^a f(x)dx = 0$$

$$\int_b^a f(x)dx = -\int_a^b f(x)dx$$

$$\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$$

The **fundamental theorem of calculus (part 1)**: If *f* is continuous on [a,b] and *F* is any antiderivative of *f* on [a,b], then $\int_a^b f(x)dx = F(x)\Big]_a^b = F(b) - F(a)$

The **fundamental theorem of calculus (part 2)**: If *f* is continuous and $F(x) = \int_a^x f(t)dt$ is an antiderivative of *f*, then $\frac{d}{dx}\left[\int_a^x f(t)dt\right] = f(x)$

Chapter 6 - Applications of Integration

To get the ∫ symbol to grow with the integrand, hold down the shift key when selecting it from the menu.

Areas are found by integrating heights. Volumes are found by integrating areas.

The **area between two curves** is either $A = \int_{left}^{right} (top - bottom)\,dx$ or

$$A = \int_{bottom}^{top} (right - left)\,dy$$

For all rotations and revolutions, the radius is always perpendicular to the axis of rotation. So, for rotation about the *x*-axis, $r = y$, and for rotation about the *y*-axis, $r = x$. The height, *h*, is always measured perpendicular to the radius or parallel to the axis of rotation. In the formulas that follow, *t* can be either axis.

In the **disk method**, the axis of rotation is one boundary of the region, and the cross-sections have an area of $A = \pi r^2$, so the volume of rotation is $V = \int_a^b \pi r^2 dt$.

In the **washer method**, the region is defined by two functions, and the cross-sectional areas are $A = \pi r_1^2 - \pi r_2^2$, so the volume of rotation is $V = \int_a^b \pi \left(r_1^2 - r_2^2\right) dt$.

In the **cylindrical shell method**, the cross-sections are cylinders, which have a lateral surface area of $A = 2\pi rh$, so the volume of rotation is $\int_a^b 2\pi rh\,dt$. This method is the only one in the chapter where the curve is a function of the *other* axis.

If you define the **arclength parameter** as $l = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}$, then the **arclength** is $L = \int_a^b l\,dt$ and the area of a surface of revolution is $S = \int_a^b 2\pi rl\,dt$

Work is found by integrating force, $W = \int_a^b F(x)\,dx$ and the **fluid force** on a surface is given by $F = \int_a^b \rho h(x) w(x)\,dx$.

Don't forget to put your name at the top of the page