Example Technology Exercise 14 James Jones

1 Problem 14.2.47

This assumption is to avoid the error Maxima gives about the upper limit of integration being real

```
(%i1) assume(x>=0,x<=3);
```

(%o1) $[x >= 0, x <= 3]$

Take 4 times the volume of the first octant

```
(%i2) 4*integrate(integrate(9-x^2-y^2,y,0,sqrt(9-x^2)),x,0,3);
```

(%o2) $\frac{81\,\pi}{2}$

Note: This problem could have also been done using polar coordinates, it's just that we hadn't covered it yet in section 14.2

```
(%i3) integrate(integrate(r*(9-r^2),r,0,3),theta,0,2*%pi);
```

(%o3) $\frac{81\,\pi}{2}$

2 Problem 14.4.36

Define the density

```
(%i4) rho:k*x*y;
```

(%o4) $k\,x\,y$

Define a moment generating function M

(%i5) M(t):=integrate(integrate(t*rho,y,x^2,x),x,0,1);

(%o5) $$\mathrm{M}\left(t\right):=\int_{0}^{1}\int_{x^2}^{x} t\,\rho\,\mathrm{d}y\,\mathrm{d}x$$

Find the mass

(%i6) mass:M(1);

(%o6) $$\frac{k}{24}$$

Find the moments of inertia

(%i7) Ix:M(y^2);
Iy:M(x^2);
I0:Ix+Iy;

(%o7) $$\frac{k}{60}$$

(%o8) $$\frac{k}{48}$$

(%o9) $$\frac{3\,k}{80}$$

Find the radius of gyrations

(%i10) xbar:sqrt(Iy/mass);
ybar:sqrt(Ix/mass);

(%o10) $$\frac{1}{\sqrt{2}}$$

(%o11) $$\frac{\sqrt{2}}{\sqrt{5}}$$

3 Problem 14.5.17

```
Define the surface
Solve it for z
Take the positive square root,
  but realize we'll need to double the answer
```

(%i12)
```
x^2+y^2+z^2-25;
solve(%,z);
f:rhs(%[2]);
```

(%o12) $z^2+y^2+x^2-25$

(%o13) $[z=-\sqrt{-y^2-x^2+25},z=\sqrt{-y^2-x^2+25}]$

(%o14) $\sqrt{-y^2-x^2+25}$

```
Formula for surface area
```

(%i15)
```
sqrt(1+diff(f,x)^2+diff(f,y)^2);
```

(%o15) $$\sqrt{\frac{y^2}{-y^2-x^2+25}+\frac{x^2}{-y^2-x^2+25}+1}$$

```
 Change over to polar coordinates
```

(%i16)
```
SA:sqrt(1+r^2/(25-r^2));
```

(%o16) $$\sqrt{\frac{r^2}{25-r^2}+1}$$

(%i17)
```
2*integrate(integrate(r*SA,r,0,3),theta,0,2*%pi);
```

(%o17) $20\,\pi$

4 Problem 14.8.23

```
Define problem
```

(%i18)
```
f:(x+y)*exp(x-y);
```

(%o18) $\left(y+x\right)\,\%e^{x-y}$

```
Define substitutions
From the sketches, we make these substitutions
  u=x-y, 0<=u<=4
  v=x+y, 4<=v<=8
```

```
Find the Jacobian
Since we're not solving for x and y
  in terms of u and v, we need the reciprocal
```

(%i19)
```
jacobian([x-y,x+y],[x,y]);
J:1/determinant(%);
```

(%o19) $$\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$$

(%o20) $$\frac{1}{2}$$

```
Change f(x,y) into g(u,v)
```

(%i21)
```
g:subst([x-y=u,x+y=v],f);
```

(%o21) $$\%e^{u}\, v$$

```
Now integrate, remember to multiply by the
absolute value of the Jacobian.
```

(%i22)
```
integrate(integrate(g*abs(J),u,0,4),v,4,8);
```

(%o22) $$12\left(\%e^{4}-1\right)$$