# Math 160: Mathematical Notation

**Purpose**:
One goal in any course is to properly use the language of that subject. Finite Math is no different and may often seem like a foreign language. These notations summarize some of the major concepts and more difficult topics of the unit. Typing them helps you learn the material while teaching you to properly express mathematics on the computer. Part of your grade is for *properly* creating and using mathematical content.

**Instructions**:
Use Word or WordPerfect to recreate the following documents. Each article is worth 10 points and should be emailed to the instructor at james@richland.edu. This is not a group assignment, each person needs to create and submit their own notation.

Type your name at the top of each document. Include the title as part of what you type.

For expressions or equations, you should use the equation editor in Word or WordPerfect. Note that the equation editor in recent versions of Microsoft Word is not as powerful as older versions. You may want to use Insert / Object / Microsoft Equation 3.0 instead. Another option is to install MathType and use it.

If there is an equation, put both sides of the equation into the same equation editor box instead of creating two objects. Be sure to use the proper symbols, there are some instances where more than one symbol may look the same, but they have different meanings and don't appear the same as what's on the assignment. There are some useful tips on the website at http://people.richland.edu/james/editor/

**If you fail to type your name on the document, you will lose 1 point**. Don't type the hints or reminders that appear on the pages.

Include both chapters for an exam in the same document. The notations are due before the exam over that material. Late work will be accepted but will lose 20% of its value per class period. No late work will be accepted after the final exam.

# Chapter 3 - Finance

**Simple Interest**
One payment, interest is not compounded

$$I = PRT$$

I = Interest, P = Principal, R = Rate, T = Time

**Compound Interest**
One payment, interest is compounded

$$A = P(1+i)^n$$

A = Amount, P = Principal, i = Periodic rate, n = Number of periods

**Future Value Annuities**
A series of payments where the balance grows in value over time

$$FV = PMT\left(\frac{(1+i)^n - 1}{i}\right)$$

FV = Future value, PMT = Payment, i = Periodic rate, n = Number of periods

**Present Value Annuities**
A series of payments where the balance decreases in value over time

$$PMT = PV\left(\frac{i}{1-(1+i)^{-n}}\right)$$

PV = Present value, PMT = Payment, i = Periodic rate, n = Number of periods

## Chapter 4 - Systems of Equations, Matrices

Operations that produce row equivalent matrices
1. Switch two rows of a matrix
2. Multiply a row by a non-zero constant
3. Add a constant multiple of one row to another row

Augmented matrix in reduced row-echelon form

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & -3 \end{array}\right] \Rightarrow \begin{array}{l} x_1 = 2 \\ x_2 = 4 \\ x_3 = -3 \end{array}$$

Matrix Multiplication

$$\begin{bmatrix} 3 & -1 & 2 \\ 4 & 7 & -5 \end{bmatrix} \begin{bmatrix} 6 & 1 \\ 2 & -5 \\ -3 & 2 \end{bmatrix} = \begin{bmatrix} 10 & 12 \\ 53 & -41 \end{bmatrix}$$

Solving a system of linear equations using matrix inverses

$$\mathbf{AX} = \mathbf{B} \Rightarrow \mathbf{X} = \mathbf{A}^{-1}\mathbf{B}$$

**Leontief Input-Output Model**

$$\mathbf{X} = \left(\mathbf{I} - \mathbf{M}\right)^{-1} \mathbf{D}$$

**X** = Output matrix
**M** = Technology Matrix
**D** = Demand Matrix

## Chapter 5 - Linear Inequalities

Hint, place following two systems into a matrix without brackets.  Choose Format / Define Spacing and set the matrix row spacing to be 120% and the matrix column spacing to be 50%.

System of linear inequalities

$$\begin{matrix} 3x & + & 4y & \le & 36 \\ 3x & + & 2y & \le & 30 \\ & & x & \ge & 0 \\ & & y & \ge & 0 \end{matrix}$$

**Existence of Solutions to a Linear Programming Problem**

- For a bounded feasible region, the objective function will always have both a maximum and minimum value of the objective function.
- For an unbounded feasible region with positive coefficients of the objective function, there will be a minimum but no maximum value.
- If the feasible region is empty, then the objective function has no maximum or minimum value.

**Fundamental Theorem of Linear Programming**

If there is a solution to a linear programming problem, then it will occur at one or more corner points of the feasible region or on the boundary between two corner points.

Although usually not written in the problem itself, almost every story problem has non-negativity constraints.  These state that the variables cannot be negative and are written as $x \ge 0$ and $y \ge 0$. These non-negativity constraints limit us to the first quadrant.

## Chapter 6 - Linear Programming

A standard maximization problem requires all problem constraints to be in the form of $\le$ a non-negative constant, but the objective function coefficients can be any real number.

A standard minimization problem requires all problem constraints to be in the form of $\ge$ any real number, but the objective function coefficients cannot be negative.

Hint, place following two systems into a matrix without brackets. Choose Format / Define Spacing and set the matrix row spacing to be 120% and the matrix column spacing to be 50%.

Standard maximization **problem**

Maximize $P = 100x_1 + 300x_2 + 200x_3$

subject to
$$\begin{matrix} x_1 & + & x_2 & + & x_3 & \le & 100 \\ 40x_1 & + & 20x_2 & + & 30x_3 & \le & 3200 \\ x_1 & + & 2x_2 & + & x_3 & \le & 160 \\ x_1 & , & x_2 & , & x_3 & \ge & 0 \end{matrix}$$

**Initial system** for a standard maximization problem after adding slack variables.

Maximize $P = 100x_1 + 300x_2 + 200x_3$

subject to
$$\begin{matrix} x_1 & + & x_2 & + & x_3 & + & s_1 & & & & & = & 100 \\ 40x_1 & + & 20x_2 & + & 30x_3 & & & + & s_2 & & & = & 3200 \\ x_1 & + & 2x_2 & + & x_3 & & & & & + & s_3 & = & 160 \\ x_1 & , & x_2 & , & x_3 & , & s_1 & , & s_2 & , & s_3 & \ge & 0 \end{matrix}$$

Be sure to reset the matrix row spacing to 150% and the matrix column spacing to 100%.

**Initial tableau** after moving the objective function to the left side of the equation.

$$\left[\begin{array}{ccccccc|c} 1 & 1 & 1 & 1 & 0 & 0 & 0 & 100 \\ 40 & 20 & 30 & 0 & 1 & 0 & 0 & 3200 \\ 1 & 2 & 1 & 0 & 0 & 1 & 0 & 160 \\ \hline -100 & -300 & -200 & 0 & 0 & 0 & 1 & 0 \end{array}\right]$$

Remember to put your name at the top of the page.

## Chapter 7 - Logic, Sets, and Counting

Truth Tables

| $p$ | $q$ | and $p \wedge q$ | or $p \vee q$ | conditional $p \to q$ | converse $q \to p$ | contrapositive $\neg q \to \neg p$ |
|---|---|---|---|---|---|---|
| T | T | T | T | T | T | T |
| T | F | F | T | F | T | F |
| F | T | F | T | T | F | T |
| F | F | F | F | T | T | T |

If **A** = { 1, 2, 4, 6 } and **B** = { 2, 3, 5 }, then ...

... the union of the sets is $\mathbf{A} \cup \mathbf{B} = \{1, 2, 3, 4, 5, 6\}$

... the intersection of the sets is $\mathbf{A} \cap \mathbf{B} = \{2\}$

**Fundamental Counting Principle**

The total number of ways that two events can happen is found by multiplying together the number of ways that each event can happen.

**Permutations**

A permutation is an arrangement of objects without repetition but with regard to order.

$${}_nP_r = \frac{n!}{(n-r)!}$$

**Combinations**

A combination is an arrangement of objects without repetition and without regard to order.

$${}_nC_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}$$

Remember to put your name at the top of the page.

# Chapter 8 - Probability

**Probability formulas**

Addition Rule

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Multiplication Rule

$$P(A \cap B) = P(A)P(B \mid A)$$

Conditional Probability

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

Complement of an Event

$$P(E') = 1 - P(E)$$

**Expected value (mean or average)**

The expected value is found by multiplying values by their probabilities and then adding.

$$E(x) = \sum xp$$

**Decision Theory**

**Expected value (Bayesian) criterion**. Find the expected value under each action and choose the action with the largest expected value.

**Maximax criterion**. Find the maximum payoff under each action and then choose the action with the largest best case scenario.

**Maximin criterion**. Find the minimum payoff under each action and then choose the action with the largest worst case scenario.

**Minimax criterion**. Find the opportunistic loss for each state of nature. Then find the maximum opportunistic loss for each action and choose the action with the smallest maximum loss.

## Chapter 9 - Markov Chains

**S** is a state matrix, **P** is the transition matrix. Both are probability matrices, meaning the sum of each row is 1.

**Regular Markov chains**

$\mathbf{S}_0$ is the initial state matrix. It reflects the beginning conditions.

$$\mathbf{S}_1 = \mathbf{S}_0\mathbf{P}$$

$$\mathbf{S}_2 = \mathbf{S}_1\mathbf{P} = \mathbf{S}_0\mathbf{P}^2$$

$$\mathbf{S}_3 = \mathbf{S}_2\mathbf{P} = \mathbf{S}_0\mathbf{P}^3$$

Steady State Matrix

$$\mathbf{S} = \mathbf{SP}$$

If the Markov chain is regular, then there is a limiting matrix $\overline{\mathbf{P}} = \mathbf{P}^{\infty}$ where each row is the Steady State matrix.

**Absorbing Markov Chains**

$$\mathbf{P} = \left[\begin{array}{c|c} \mathbf{I} & \mathbf{0} \\ \hline \mathbf{R} & \mathbf{Q} \end{array}\right]$$

Fundamental Matrix **F** (expected frequencies)

$$\mathbf{F} = \left(\mathbf{I} - \mathbf{Q}\right)^{-1}$$

The element in row R, column C of the fundamental matrix represents the expected number of times you will spend in state transient C of the system before ending up at some absorbing state if you start in transient state R.

Limiting Matrix (long term probabilities)

$$\overline{\mathbf{P}} = \mathbf{P}^{\infty} = \left[\begin{array}{c|c} \mathbf{I} & \mathbf{0} \\ \hline \mathbf{FR} & \mathbf{0} \end{array}\right]$$

The element in row R, column C of the matrix **FR** represents the long term probability of ending up in absorbing state C if you started in transient state R.

## Chapter 10 - Game Theory

If the game matrix is $\mathbf{M} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ and $D = (a+d) - (b+c)$, then the solution to a two player, zero-sum, non-strictly determined game is

$$\mathbf{P}^* = \begin{bmatrix} \dfrac{d-c}{D} & \dfrac{a-b}{D} \end{bmatrix} \quad \mathbf{Q}^* = \begin{bmatrix} \dfrac{d-b}{D} \\ \dfrac{a-c}{D} \end{bmatrix} \quad v = \frac{ad-bc}{D}$$

**Linear Programming Problem**

Assume that $\mathbf{M} = \begin{bmatrix} a & b \\ c & d \\ e & f \end{bmatrix}$ has all positive entries (add a constant if it doesn't).

The optimal row player strategy $\mathbf{P}^* = \begin{bmatrix} p_1 & p_2 & p_3 \end{bmatrix}$ is found by solving:

Minimize $z = \dfrac{1}{v} = x_1 + x_2 + x_3$ where $x_1 = \dfrac{p_1}{v}$, $x_2 = \dfrac{p_2}{v}$, and $x_3 = \dfrac{p_3}{v}$

Subject to
$$\begin{array}{ccccccc} ax_1 & + & cx_2 & + & ex_3 & \geq & 1 \\ bx_1 & + & dx_2 & + & fx_3 & \geq & 1 \\ x_1 & , & x_2 & , & x_3 & \geq & 0 \end{array}$$

The optimal column player solution $\mathbf{Q}^* = \begin{bmatrix} q_1 \\ q_2 \end{bmatrix}$ is found by solving:

Maximize $z = \dfrac{1}{v} = y_1 + y_2$ where $y_1 = \dfrac{q_1}{v}$ and $y_2 = \dfrac{q_2}{v}$

Subject to
$$\begin{array}{ccccc} ay_1 & + & by_2 & \leq & 1 \\ cy_1 & + & dy_2 & \leq & 1 \\ ey_1 & + & fy_2 & \leq & 1 \\ y_1 & , & y_2 & \geq & 0 \end{array}$$