<u>Presidential Election 1886</u>

Democrat Grover Cleveland versus Benjamin Harrison

Cleveland – 5,540,309

Harrison – 5,439,853

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Difference of 100,456

Electoral College

Cleveland – 168

Harrison - 233

PRESIDENTIAL ELECTION 2000

105 MILLION VOTES CAST

Bush – 550,000 fewer popular votes

Electoral College

Bush – 271

Gore – 266

What did founding fathers hope with this system?

- **Electoral College would discourage party politics**

Yet protect individual states from excessive federal power.

Each state has as many electors as it has congressional seats.

States electoral vote depends on the size of its House delegation – roughly proportional to population, modified by adding 2 votes corresponding to 2 Senate seats.

1990 –
Kentucky – 3.7 million – 8 electoral votes

Idaho - 4 electoral votes, but 27% of Kentucky's population. (999,000)

Do Small states have electoral power disproportionate to their populations?

Why not a completely fair apportionment method?

Plurality Method

Each voter gives one vote to his or her top-ranked candidate. The candidate with the most votes, a plurality of the votes wins the election.

Most common method

Example – 100 votes

33 votes for Alternative a
31 votes for Alternative b
36 votes for Alternative c

Alternative c does not have a majority of the votes; it does have a plurality of the votes.

- **Therefore, Alternative c is declared the people's choice in spite of not being the choice of 64 of 100 voters.**

Example of Plurality

Only the top ranked candidates matter!

Banquet – choices of beef = B, chicken = C, or pork = P

#voters	3	3	4	2
1st	A	C	B	C
2nd	B	A	A	B
3rd	C	B	C	A

Only B matters. The winner is beef because it receives a plurality of the votes.

Example of Plurality
School board election – 12 voters
Alice = A, Bob = B and Cathy = C

#VOTERS	3	3	4	2
1ST	A	C	B	C
2nd	B	A	A	B
3rd	C	B	C	A

Cathy gets 3 + 2 for 5 votes

Cathy is the elected chairperson because of a plurality of the votes.

Pairwise Comparison method

Each votes gives a complete ranking of the candidates. For each pair of candidates and b, the number of voters preferring a is compared with the number of votes preferring b. The candidate receiving more votes is awarded on point. If the two candidates receive an equal number of votes, each is awarded a half point. The candidate with the most points is the winner of the election.

Pairwise difficult with more than 4 candidates.

If n candidates, the number of comparisons is C(n,2)

$$C = \frac{n(n-1)}{2}$$

Example: 4 candidates taken 2 at a time

$$d = \frac{total\ population}{possible\,\#\ of\ \ representatives}$$

$$c = \frac{n(n-1)}{2} = \frac{4(4-1)}{2} = \frac{4(3)}{2} = 6$$

Example: 5 candidates

$$c = \frac{n(n-1)}{2} = \frac{5(5-1)}{2} = \frac{5(4)}{2} = 10$$

Banquet – choices of beef = B, chicken = C, or pork = P

#votes	6	5	4
1^{st}	B	C	P
2^{nd}	P	P	C
3^{rd}	C	B	B

N = 3 candidates

$$C(n,2) = C\ (3,2) = \frac{3(3-2)}{2} = 3$$

3 pairs examined

P and C
P and B
C and B

Banquet decision by Pairwise

Pork and Chicken → P and C

pork higher 6 + 4 = 10

chicken higher 5 voters

pork over chicken by margin of 10 to 5 and pork receives one point

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Pork and Beef → P and B

pork higher 5 + 4 = 9

beef gets 6 votes

pork over beef margin of 9 to 6 and pork receives one point

Chicken over beef 9 votes

Beef higher than chicken 6 votes

Chicken over beef margin of 9 to 6 and chicken gets one point

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Pork has 2 points, chicken 1 point, beef 0 points

Winner is pork

BORDA METHOD

***Each voter must give complete ranking of candidates.**
***N is number of candidates.**
*** Each first place vote a candidate receives is worth n points. Second place is n -1votes, third place is n-2 votes, etc. and last place gets 1 point.**

***Highest tally of points is winner.**

Winner of popular music awards.
Heisman Trophy

Borda Method

Banquet – choices of Beef = B
Chicken = C
Pork = P

#votes	6	5	4
1^{st}	B	C	P
2^{nd}	P	P	C
3^{rd}	C	B	B

Beef = (6x3) + (5+4)x1=18 + 9 = 27

Chicken = (5x3) + (4x2) +(6x1 =
15 + 8 + 6 = 29

Pork = (4x3) + (6 + 5)x2 = 12 + 22 = 34

Pork is the Winner

Hare Method

- **Variation of plurality method**
- **Candidates eliminated in sequential rounds of voting.**

- **President of France done this way**

- **Officials with Australia**

- **Also known as plurality with elimination method or the single transferable vote system.**

- **No ranking**

- **Candidate receives a majority of the votes, it is declared the winner. Rounds continue until winner.**

Plurality with Elimination

Banquet Decision

#votes	6	5	4
1st	B	C	P
2nd	P	P	C
3rd	C	B	B

Beef gets 6 votes
Chicken gets 5 votes
Pork gets 4 votes
Majority needs 8 votes, pork eliminated

#votes	6	5	4
1st	B	C	C
2nd	C	B	B

Beef gets 6 votes
Chicken gets 9 votes
Chicken gets majority – winner

Approval Voting Method

- **Each voter is allowed to give one vote to as many candidates as he or she wishes.**
- **The candidate with the most approval votes is the winner of the election.**

# of Voters	Ranking	Number of voters voting for 2 of the three candidates
6	b>p>c	x, $0 \leq x \leq 6$
5	c>p>b	y, $0 \leq y \leq 5$
4	p>c>b	z, $0 \leq z \leq 4$

Beef always gets 6 approval votes

No additional votes for beef.

Chicken always gets 5 votes.

Can get up to 4 additional votes

Pork gets 4 votes and x = 6 and y = 5 votes from 11 voters with top two rankings.

Beef = 6

Chicken 5 + z

Pork 4 + x + y

If each voter distrust the new system and votes only for his or her top-ranked candidate – plurality method
Beef wins

If 3 of 6 voters with the top ranking decide to approve of pork and beef, then x = 3 additional votes go to pork
Beef gets 6 approval votes
Chicken gets 5 approval votes
Pork gets 4 + 3 = 7 votes
Pork wins

Beef gets 6 votes
Chicken gets 5 + 3 = 8 votes
Pork gets 4 + 2 + 1 = 7 votes
Chicken wins

Capricious nature of approval voting. Changed how many candidates received an approval vote.

SEQUENTIAL PAIRWISECOMPARISON METHOD

- **Entire field compared two at a time**
- **Predetermined order**
- **Candidate with fewer votes is eliminated – more votes advances.**
- **Process continues until two candidates.**

Site of intergalactic Meeting

Saturn, Jupiter, Mars, Saturn, Venus
Our group wants Mars to win!!! –
Polling show the following:

- **Saturn beats Jupiter**
- **Saturn beats Mars**
- **Venus beats Saturn**
- **Jupiter beats Mars**
- **Jupiter beats Venus**
- **Mars beats Venus**

Order s, j, m, v

Venus wins

Order j,m,v,s

Mars out on first vote
Saturn wins

Order v,s,m,j

----Jupiter wins

Order s,j,v,m

--- Mars wins

Method used by U.S. Congress for amendments to bills

2000 Presidential Election
Palm Beach, Florida

Potential 2000 voters confused by "butterfly" ballot may have voted for reform candidate Pat Buchnan instead of Al Gore

Bush won by 537 votes Florida thus won 25 electoral votes.

Note votes were misrecorded or miscounted in other counties

Only about 51% of voting age population actually voted.

Determining the will of the people is elusive.

4 Desirable attributes for any voting method:

1. **Majority Criterion**

2. **Condorcet criterion**

3. **Monotonicity Criterion**

4. **Independence of irrelevant alternatives criterion**

   (1) and (2) desirable qualities for a voting method when it is used a single time

   (2) and (4) desirable when method is to be used twice in election process

MajorityCriterion

If a candidate has a majority of the first place rankings in a voter profile, then that candidate should be the winner of the election.

Example
Banquet decision by Borda Method

#votes	6	5	4
1^{st}	B	C	P
2^{nd}	P	P	C
3^{rd}	C	B	B

Beef = (6x3) + (5+4)x1=18 + 9 = 27

Chicken = (5x3) + (4x2) +(6x1 =
15 + 8 + 6 = 29

Pork = (4x3) + (6 + 5)x2 = 12 + 22 = 34

Pork is the Winner

Pork is winner

Fails Majority Criterion

Condorcet Criterion

If a Condorcet canditate exists for a profile, then the Condorcet candidate should be the winner of the election.

A candidate who can win a pairwise comparison with every other candidate is called a Condorcet candidate.

Monotonicity Criterion

If Candidate x wins an election and. Before a selection, the voters who rearrange their rankings move Candidate x to the top of their rankings, then Candidate x should win the second election.

The plurality method respects the monotonicity criterion

Note that the Borda method and the pair wise comparison method can fail the monotonicity criterion.

How the voter shifts the other candidates is not restricted.

The Independence of Irrelevant Alternatives (IIA) Criterion

If Candidate x wins a first election and one or more of the losing alternatives drops out before a second vote, the winner x of the first election should win the second election.

Banquet Decision

#votes	6	5	4
1^{st}	B	C	P
2^{nd}	P	P	C
3^{rd}	C	B	B

Beef is selected by plurality method.

If second election, pork is removed. The former pork supporters vote for chicken and chicken wins.

If chicken removed, then pork would win.

Borda method can fail as well as pairwise to meet Criterion

There does not exist and never will exist any voting method that simultaneously satisfies the majority criterion, the Condorct criterion, the monotonicity criterion, and the independence of irrelevant alternatives criterion.

Arrow's Impossibility Theorem.

A contradictory profile of voters will exist for any new voting system yet to be invented.

>Compute the standard divisor

d = (total population) /(total number of seats)

or

$$d = \frac{total\ population}{possible\#\ of\ \ representatives}$$

>Compute the standard quota for each state

Q = state’s population /d

Possibilities of Apportionment

The Hamilton Method

>Compute the standard divisor

>Compute the standard quota for each state

> Give any additional seats one at a time (until no seats are left) to the states with the largest fractional parts of their standard quotas.

Possibilities of Apportionment

The Jefferson Method

Compute md, the modified divisor

Compute mQ, the modified quotient for each state.

MQ = state population /md

Round each state's modified quoto mQ down ot the nearest integer.

Give each state this integer number of seats.

(md) is complicated to obtain

The Webster Method

> Compute md, the modified divisor.
> Compute mQ, the modified quota
  mQ = (state's population/md)

>Round each state's modified quota mQ up to the nearest integer if it fractional part is greater than or equal to 0.5 and down if the nearest integer if its fractional part is less than 0.5

> Give each state this integer number of seats

(md) harder still to calculate

Adams Method

John Quincy Adams – sixth President

Never adopted

Compute md, the modified divisor

Compute mQ, the modified quota for each state

Round each state’s modified quota mQ up to the nearest integer.

Give each state this integer number of seats

Hill-Huntington Method
Currently used
Replaced Webster method in 1941

Mathematics complicated

- **compute md, the modified divisor**
- **compute mQ, the modified quota**
- **Round each state's modified quota mQ to the nearest integer using the Hill-Huntington rounding scheme**
- **Give each state this integer number**

Geometric mean is the square root of the product of the two numbers

Hill Huntington – uses the geometric mean of a = the integer part of a modified quota and b = the integer part of the modified quota + 1 as the cutoff point for rounding the modified quota up to the nearest integer, instead of down to the nearest integer

Example
State has mQ = 6.49
Find the geometric mean of 6 and
6 + 1= 7 -→ 6.481

value mQ = 6.49 which is greater than 6.481

So round up to 7

Apportionment

Quota Rule

The integer number of objects apportioned to each state must be either the standard quota Q rounded down to the nearest integer, or the standard quota Q rounded up to the nearest integer.

Jefferson method favored large states. Jefferson method, apportioned more seats to a state than the states value of Q rounded up to the nearest integer.

Hamilton method satisfies quota rule. Would be the method of choice if only benchmark was quota rule

Jefferson method and Webster method fails quota rule because they use modified divisors, not the standard divisor.

The Alabama Paradox

The situation in which an increase in the number of objects being apportioned actually forces a state to lose one of those objects is known as the Alabama paradox.

Following 1880 census
Increasing House size to 299 seats – Alabama would receive 8 seats. If house size was 300, then Alabama would receive 7 seats.

Population Paradox

Occurs when, based on updated population figures, a reapportionment of a fixed number of seats causes a state to lose a seat to another state, although the percent increase in the population of the state that loses the seat is bigger than the percent increase in the population of the state that gains the seat.

Discovered about 1900

The New States Paradox

Occurs when a reapportionment of an increased number of seats, necessary due to the addition of a new state, causes a shift in the apportionment of the original states.

Occurred in 1907 when Oklahoma joined the Union as the 46^{th} state.

Historic note
Following 1870 census.
283 seats divided
Official apportionment done by Hamilton method and agreed with Webster method

Power grab on floor
292 seats
Believed to be unconstitutional

Gave Rutherford Hays enough electoral votes to become president.

Tileden won a popular majority and would have won electoral votes if done using Hamilton method

Balinski and Young Impossibility Theorem

Any apportionment method that always satisfies the quota rule, will by its nature, permit the possibility of a paradoxical apportionment. Likewise, and apportionment method that does not permit the possibility of a paradoxical apportionment will, by nature, fail to always satisfy the quota rule.

<u>Ultimate method is impossible!!!</u>